TEST 2 REVIEW

PART 1 - Computer Part

<u>Regarding the use of Maple for the exam</u> – Due to the immense capabilities of Maple in the area of applications of integrals, the use of this software will be limited on this test.

- **Most of the application problems will occur on Part 2 of the exam.**
- **You must know the formula for each application and how to set up the definite integral on Part 2 of the exam, but not necessarily evaluate it.**
- **You may be asked to evaluate a definite integral that is not too complicated on Part 2, but the focus will be on your ability to set up the appropriate formula.**
- **On the computer part of the exam, you can expect to see a few applications where either the function cannot be graphed, the equation cannot be solved, or the integral cannot be completed by hand. But these examples will be limited.**

Textbook:

Page 488 Exercises 6, 23, 25, 34

Additional Problems:

1. Consider the length of the arc of the curve defined by the parametric equations

$x(t) = e^t \cos(t)$ and $y(t) = e^t \sin(t)$ for $0 \le t \le \frac{\pi}{2}$.

a. Sketch the graph.

b. Find the arc length using integration.

2. **a.** Give a lower bound and an upper bound for the average value of $f(x) = x^3 + x + 1$ on the interval [0, 2] and sketch the graph of $f(x)$ on [0, 2].

b. Find the average value of $f(x) = x^3 + x + 1$ on the interval [0,2].

c. Find c such that $f_{ave} = f(c)$.

d. Sketch the graph of *f* and a rectangle whose area is the same as the area under the graph of *f*. What are the dimensions of the rectangle?

e. What is the area under *f*(*x*)?

3. **a.** Sketch the region bounded by the graphs of $f(x) = 0.5x^3 - 5x^2 + 15x - 15$ and $g(x) = 5\sqrt{x}$ and $x = 0$.

b. Estimate the location of the centroid.

c. Find the centroid of this region and plot the centroid to see if your answer is reasonable.

PART 2 – Non Computer Part

NOTE: In addition to completing this review sheet, review all homework and in-class exercises. You should also go over the Test 1 Review Sheet. Remember, the tests are <u>cumulative</u> and you will also be tested on the Unit 1 material, including differentiation, area and l'Hospital's Rule.

Textbook:

Page 325: Exercises 27 – 34 all.

Page 487: Concept Check 5, 6, 9
Page 426: Exercises 55, 57, 59

1. Choose from the following applications of integration and write a paragraph outlining the steps in the derivation of the formula(s). Include a sketch where appropriate.
- **a.** Volume
- **b.** Arc Length
- **c.** Average Value
- **d.** Centroids

2. Consider the length of the arc of the parabola $y = x^2 - 3x + 2$ from $x = 0$ to $x = 4$.
- **a.** Sketch the graph.
- **b.** Give a lower bound and an upper bound for the arc length.
- **c.** Set up the integral for arc length, but do not evaluate it.

3. Consider the length of the arc of the parabola $x = 4 - y^2$ in quadrant I.
- **a.** Sketch the graph.
- **b.** Give a lower bound and an upper bound for the arc length.
- **c.** Find the arc length using integration.

4. **a.** Give a lower bound and an upper bound for the average value of $f(x) = \dfrac{x^2+1}{x^2}$ on the interval [0.5, 2] and sketch the graph of $f(x)$ on [0.5, 2] (Note: you would be given this sketch on the exam).

b. Find the average value of the function over the interval.

c. Find *c* such that $f_{ave} = f(c)$.

d. Sketch the graph of *f* and a rectangle whose area is the same as the area under the graph of *f*. What are the dimensions of the rectangle?

e. What is the area under *f*(*x*)?

5. **a.** Sketch the graph of $f(x) = x^3 + 1$ on the interval [0, 2].

b. Estimate the location of the centroid.

c. Find the centroid of this region and plot the centroid to see if your answer is reasonable.

6. **a.** Sketch the region bounded by the graphs of $y = x^3,\ y = x,\ 0 \le x \le 1$.

b. Estimate the location of the centroid.

c. Find the centroid of this region and plot the centroid to see if your answer is reasonable.

INTEGRATION PRACTICE

On separate paper, evaluate each of the following by using one of the following techniques of integration:
U-Substitution
Trig Integrals
Integration by Parts
Change of Variable Substitution
Trig Substitution
Partial Fractions
Remember to think first about the standard integration formulas and possibly a simple substitution before committing to a more complex technique.

1. $\int xe^{3x}dx$

2. $\int \frac{t-6}{t^2-2t}dt$

3. $\int \tan^3(\theta)d\theta$

4. $\int x\arctan(x)dx$

5. $\int \cos^5(x)\sqrt{\sin(x)}dx$

6. $\int \frac{\sqrt{25-y^2}}{y}dy$

7. $\int \frac{3x^2+x+1}{x^2(x+1)}dx$

9. $\int e^t\sin(3t)dt$

10. $\int \frac{1}{x\sqrt{1+4x}}dx$

11. $\int \frac{dx}{\left(16+x^2\right)^{3/2}}$

12. $\int \frac{t+9}{t^3+9t}dt$

13. $\int x\left(\ln x\right)^2 dx$

14. $\int \frac{1}{\sqrt{4x^2-25}}dx$

15. $\int \frac{dx}{e^x-e^{-x}}$

8. $\int \sec^3(\theta)\tan(\theta)d\theta$

16. $\int x\sqrt{x+2}dx$

Find the following limits. Use l'Hospital's Rule where appropriate.

17. $\lim_{x\to 1}\frac{\ln(x)}{x^2-1}$

18. $\lim_{x\to 0}\frac{e^x-(1-x)}{x}$

19. $\lim_{x\to\infty}\frac{\ln(x)}{x}$

20. $\lim_{x\to 0^+}\frac{1-\cos(x)}{x^3}$

21. $\lim_{x\to\infty} e^x x^{-1}$

22. $\lim_{x\to 0^+} x^2\ln(x)$

23. $\lim_{x\to 2}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$

24. $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$

25. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

26. $\lim_{x\to 1^+}(x-1)^{\ln(x)}$

27. Explain why l'Hôspital's Rule does not apply to each of the following:

a. $\lim_{x\to 0^+}\frac{\cos(x)-x}{\sqrt{x}}$ **b.** $\lim_{x\to\infty}\frac{\sin(x)}{x^2+\pi}$ **c.** $\lim_{x\to 0^+} x^{\frac{1}{x}}$

28. If $f(x)$ is continuous on $[0,\infty)$, how is $\int_0^\infty f(x)dx$ defined?

29. What does it mean to say that $\int_{-\infty}^b f(x)dx$ converges?

For problems 30 – 34, determine whether the integral converges or diverges. If it converges, find its value.

30. $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$

31. $\int_3^\infty \frac{dx}{\sqrt{2x-1}}$

32. $\int_{-\infty}^4 \frac{dx}{(5-x)^2}$

33. $\int_0^1 \frac{dx}{(x-1)^{2/3}}$

34. $\int_{\pi/2}^{\pi}\sec(x)dx$

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ANSWERS

Part 1 – Computer Part

1.a. $(-.785902, 1.85455)$ **and** $(.785902, 1.85455)$

b. (i) **24.5571 cu. units**

(ii) **1.84838 cu. units**

2 **b.** The arc length is between 4.91332 and 6.36136.

c. $s = 5.38883$

3. **a.** The average value is between 1 and 11.

b. $f_{ave} = 4$

c. $c = 1.2134$

d. Area of rectangle=2(4) = 8 sq units

e. Area under $f(x)$:

$$A = \int_0^2 \left(x^3 + x + 1\right) dx = 8 \text{ sq units}$$

4. $(3.298988, 2.88623)$

Part 2 – Non Computer

Text Page 325:

28. 0
30. ∞
32. 0
34. 1

1. Explanation should include

a. statement of the problem

b. subdivision of interval

c. form the construction

d. do a calculation of the construction

e. write the sum

f. write the limit

2. **a.** Graph

b. $\int_0^4 \sqrt{(2x-3)^2+1}\,dx$

(Check in Maple: $s = 9.7782$)

3. a. Graph

b. $\int_0^2 \sqrt{(-2y)^2+1}\,dy \underset{trig\ subs}{=} \sqrt{17} - \frac{1}{4}\ln(\sqrt{17}-4)$

$\approx 4.6468(Maple\,check)$

4. **a.** The average value is between 1.25 and 5.

b. $f_{ave} = 2$

c. $c = 1$

d. Area of rectangle=2(3/2)=3 sq.units

e. $A = \int_{0.5}^{2} \frac{x^2+1}{x^2}\,dx = 3$

5. $\left(\frac{7}{5}, \frac{33}{14}\right)$

6. $\left(\frac{8}{15}, \frac{8}{21}\right)$

Integration Practice

1. $\frac{e^{3x}}{9}(3x-1)+C$

2. $\ln\left|\frac{t^3}{(t-2)^2}\right|+C$

3. $\frac{\tan^2\theta}{2}+\ln|\cos\theta|+C$

4. $\frac{(x^2+1)}{2}\arctan x - \frac{x}{2}+C$

5. $\frac{2}{3}(\sin x)^{3/2} - \frac{4}{7}(\sin x)^{7/2} + \frac{2}{11}(\sin x)^{11/2} + C$

6. $5\ln\left|\frac{5-\sqrt{25-y^2}}{y}\right| + \sqrt{25-y^2}+C$

7. $3\ln|x+1| - \frac{1}{x}+C$

8. $\frac{\sec^3\theta}{3}+C$

9. $\frac{e^t}{10}\left(\sin(3t)-3\cos(3t)\right)+C$

10. $\ln\left|\frac{\sqrt{1+4x}-1}{\sqrt{1+4x}+1}\right|+C = \ln\left|\frac{\left(\sqrt{1+4x}-1\right)^2}{4x}\right|+C =$

$2\ln\left|\sqrt{4x+1}-1\right| - \ln|x| + C$

11. $\frac{x}{16\sqrt{16+x^2}}+C$

12. $\frac{1}{3}\arctan\left(\frac{t}{3}\right)-\frac{1}{2}\ln\left(t^2+9\right)-\ln|t|+C$

13. $\frac{x^2(\ln x)^2}{2}-\frac{x^2(\ln x)}{2}+\frac{x^2}{4}+C$

14. $\frac{1}{2}\ln\left|\sqrt{4x^2-25}+2x\right|+C$

15. $\frac{1}{2}\ln\left|\frac{e^x-1}{e^x+1}\right|+C$

16. $\frac{2}{15}(x+2)^{3/2}(3x-4)+C$

17. 1/2

18. 2

19. 0

20. ∞ (does not exist)

21. ∞ (does not exist)

22. 0

23. $-\frac{3}{2}$

24. 1

25. e

26. 1

27. **a.** Direct substitution yields $\frac{1}{0}$, which indicates ∞

b. Direct substitution yields $\frac{\#}{\infty}$, which indicates 0.

c. Direct substitution yields 0^∞, which is not indeterminate. The expression shows a smaller and smaller number to a larger and larger power, which indicates 0.

28. $\int_0^\infty f(x)\,dx=\lim_{b\to\infty}\int_0^b f(x)\,dx$

29. $\lim_{a\to-\infty}\int_a^b f(x)\,dx$ exists

30. Converges to $\frac{2}{e}$

31. Diverges

32. Converges to 1

33. Converges to 3

34. Diverges