UNIT 3 HOMEWORK

NOTE: *Early Transcental* (ET) Sections are located at the end of the ***Concepts*** text.

ET Section 11.1 Sequences

A. EWA Homework 15

B. ET Page 684 1, 2, 3 – 9 odd; 13, 15.
For problems: 17, 21, 25, 29, 35, 39, 43, 45 use Maple to obtain numerical and graphical evidence about the convergence or divergence of the sequence but then prove your result by finding the limit analytically.
Do problems: 49, 54, 57, 63, 65.

C. Additional Problems

For Exercises 1 – 2, write enough terms to determine whether the sequence defined as follows is convergent or divergent:

1. $a_1 = 1, \quad a_2 = 3, \quad a_{n+1} = a_n - a_{n-1}$ **2.** $a_1 = 1, \quad a_{n+1} = \dfrac{2a_n}{n+1}$

For each of the sequences in 3 – 4,

a) write the next two terms in the sequence,
b) find a formula for the general term a_n, $n \geq 1$,
c) give a recursive definition that generates the same sequence and,
d) show a check that each method actually gives the required sequence by using your solutions to b) and c) to generate the first three terms of the sequence.

3. 4, 12, 20, 28, 36,… **4.** 2, 5, 10, 17, 26,…

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ANSWERS:

B.

2. A sequence $\{a_n\}$ converges if $\lim_{n\to\infty} a_n$ exist. Examples: $\left\{\frac{1}{n}\right\};\left\{\frac{3n-1}{n+2}\right\}$. A sequence diverges otherwise. Examples: $\left\{\frac{n}{\ln(n)}\right\};\left\{e^n\right\}$

54. a. List some terms: $\{1,3,1,3,1,3,\ldots\}$. The sequence diverges, b/c the limit DNE.

b. If the first term is 2 then $a_n = 2, \textit{for all } n$; this sequence converges.

C.

1.	Diverges	**2.** Converges	
3.	**a)** 44, 52	**b)** $a_n = 4 + (n-1)8, n \geq 1$	**c)** $a_1 = 4,\ a_{n+1} = a_n + 8$
4.	**a)** 37, 50	**b)** $a_n = n^2 + 1, n \geq 1$	**c)** $a_1 = 2,\ a_{n+1} = a_n + 2n + 1$

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ET Section 11.2 Series

A. EWA Homework 16

B. ET Page 694 Do 1, 2;

For problems: 3 – 8 all, follow the directions in Supplement 11.2 to use Maple to generate the first 50 terms of the sequence of partial sums. Then answer the questions in the text.
Do 9; for problems: 11 – 29 odd, use what you know about geometric series, harmonic series and the nth term Test for Divergence to determine convergence or divergence.
Do 37, 53.

C. Additional Problems

1. Suppose $\sum a_n$ converges and the sum is *s*. What is $\lim\limits_{n\to\infty} s_n$? What is $\lim\limits_{n\to\infty} a_n$?

2. Suppose $\lim\limits_{n\to\infty} a_n = 1$. What can you say about $\sum a_n$?

For Exercises 3 – 4, determine whether the series is convergent or divergent. If it is convergent, find its sum.

3. $\sum\limits_{n=4}^{\infty} 3\left(\frac{5}{8}\right)^n$ **4.** $\sum\limits_{n=5}^{\infty} 2\left(-\frac{3}{4}\right)^n$

D. Integration Review: Evaluate the following integrals.

1. $\int e^{\sqrt{x}}\,dx$ **2.** $\int \sqrt{\cos(x)}\sin(x)\,dx$ **3.** $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

4. $\int \frac{2x+4}{x^3-2x^2}\,dx$ **5.** $\int \frac{3x^2+2}{x^3+2x-8}\,dx$

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Answers:

B.

2. The sum of the series is 5, i.e. the sequence of partial sums converges to 5.

4. Graph on Maple. The sequence of partial terms appears to diverge. The partial sums appear to increase without bound. Note: $\lim\limits_{n\to\infty} a_n = 2 \neq 0$.

6. Graph on Maple. The sequence of partial sums appears to converge. The graph of the partial sums appears to level out and converge to 2.5. Note: This is geometric series with $|r = .6| < 1$.

8. Graph on Maple. The sequence of partial sums appears to converge. The graph appears to level off, however 10 points is not a sufficient amount of data to look at.

C.

1. $\lim\limits_{n\to\infty} s_n = s$, $\lim\limits_{n\to\infty} a_n = 0$

2. If $\lim\limits_{n\to\infty} a_n \neq 0$ or does not exist, then the series $\sum\limits_{n=1}^{\infty} a_n$ is divergent.

3. $\sum_{n=0}^{\infty} 3\left(\frac{5}{8}\right)^n = \frac{3}{1-(5/8)} = 8$

$\sum_{n=4}^{\infty} 3\left(\frac{5}{8}\right)^n = 8 - \left(3 + \frac{15}{8} + \frac{75}{64} + \frac{375}{512}\right) = \frac{625}{512}$

4. $\sum_{n=0}^{\infty} 2\left(-\frac{3}{4}\right)^n = \frac{2}{1-\frac{-3}{4}} = \frac{8}{7}$

$\sum_{n=5}^{\infty} 2\left(-\frac{3}{4}\right)^n = \frac{8}{7} - \left(2 - \frac{3}{2} + \frac{9}{8} - \frac{27}{32} + \frac{81}{128}\right) = -\frac{243}{896}$

D. 1. $2e^{\sqrt{x}}\left(\sqrt{x}-1\right)+C$ 2. $-\frac{2}{3}\cos^{3/2}(x)+C$ 3. $2e^{\sqrt{x}}+C$

4. $-2\ln|x| + \frac{2}{x} + 2\ln|x-2| + C$ **5.** $\ln\left|x^3+2x-8\right|+C$

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ET Section 11.3 The Integral Test and Estimates of Sums

A. EWA Homework 17

B. ET Page 703 3 – 13 odd, 17, 21, 23

C. Additional Problems

For Exercises 1 – 7, determine whether the series is convergent or divergent. State your reasoning.

1. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

2. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \ldots$

3. $\sum_{n=1}^{\infty} \frac{1}{n^{1.01}}$

4. $\sum_{n=1}^{\infty} \sqrt{\frac{1}{n}}$

5. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$

6. $\sum_{n=1}^{\infty} \frac{1}{n^e}$

7. $\sum_{n=1}^{\infty} n^{-0.99}$

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Answers:

C.

1. p=3, converges
2. p=3/2, converges
3. $p=1.01$, converges
4. $p=1/2$, diverges
5. $p=4/3$, converges
6. $p=e$, converges
7. $p=0.99$, diverges

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ET Section 11.4 The Comparison Tests

A. EWA Homework 17

B. ET Page 709 3, 5, 7, 9, 17, 19, 21, 23

C. Additional Problems
Integration Review: Evaluate the following integrals.

1. $\int \arcsin(x)\,dx$ **2.** $\int_0^{\frac{\pi}{2}} \tan(x)\,dx$ **3.** $\int_{-\infty}^{0} \frac{e^x}{3-2e^x}\,dx$

4. $\int \frac{3x^4+3x^3-5x^2+x-1}{x^2+x-2}\,dx$ **5.** $\int \frac{dx}{\left(3+x^2\right)^{3/2}}$

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Answers:
C.

1. $x\arcsin(x)+\sqrt{1-x^2}+C$ 2. ∞ **, diverges** 3. $\frac{1}{2}\ln 3$**, converges**

4. $x^3+x-\frac{1}{3}\ln|x+2|+\frac{1}{3}\ln|x-1|+C$ **5.** $\frac{x}{3\sqrt{3+x^2}}+C$

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ET Section 11.5 Alternating Series

A. EWA Homework 18

B. ET Page 713 1 – 13 odd, 23, 25, 27, 29, 32, 33

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Answers:

B. 32. $p>0$, the series converge by AST. If $p>1$, the series converge absolutely.

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ET Section 11.6 Absolute Convergence and the Ratio and Root Tests

A. EWA Homework 18

B. ET Page 719 1, 3, 5, 7, 8, 9, 18, 21, 23, 25, 31

C. Additional Problems
Integration Review: Evaluate the following integrals.

1. $\int t\,\sqrt[3]{t-4}\,dt$ **2.** $\int x\tan^2\left(x^2\right)\sec^2\left(x^2\right)dx$ **3.** $\int_a^{+\infty} \frac{xdx}{\left(x^2+1\right)^2}$

4. $\int \frac{x^3+3x^2+x+4}{\left(x^2+1\right)\left(x^2+3\right)}dx$ **5.** $\int \left(x^3+1\right)\cos(x)\,dx$

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Answers

B.

8. Divergent by Ratio test

18. Divergent by the Ratio Test.

C.

1. $\frac{3}{7}(t-4)^{4/3}(t+3)+C$ **2.** $\frac{1}{6}\tan^3(x^2)+C$ **3.** $\frac{1}{2(a^2+1)}$, converges

4. $\frac{1}{2}\arctan(x)+\frac{1}{2}\ln(x^2+3)+\frac{5}{2\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right)+C$ **5.** $(x^3+1)\sin(x)+3x^2\cos(x)-6x\sin(x)-6\cos(x)+C$

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ET Section 11.7 Strategies for Testing Series

A. ET Page 722 1 – 10 all, 13, 16, 21, 28, 31

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Answers:

A.

2. Convergent by Root Test
4. Conditionally convergent by AST
6. Divergent by comparison to harmonic series (or integral test)
8. Divergent by Ratio Test
10. Convergent by integral test
16. Divergent by LCT to $\sum_{n=1}^{\infty}\frac{n^2}{n^3}=\sum_{n=1}^{\infty}\frac{1}{n}$
28. Convergent by integral test

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ET Section 11.8 Power Series

A. EWA Homework 19

B. ET Page 727 1 – 11 all, 15, 19, 22, 30

C. Additional Problems
Integration Review: Evaluate the following integrals.

1. $\int_0^1 \ln(x)\,dx$ **2.** $\int \frac{x^3}{\sqrt{1-x^8}}dx$ **3.** $\int_4^8 \frac{\sqrt{x-4}}{x}dx$

4. Integrate $\int \frac{x^2}{(1+x^2)^2}dx$ using the method of: **a)** trig substitution **b)** integration by parts.

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Answers:

B.

2. The radius of convergence of a power series is the value of R, such that the power series converges when $|x-a|<R$. R is found using the Ratio test. The interval of convergence is the interval on which the series converges. It is found by solving the above inequality for *x.*

4. $\lim_{n\to\infty}\left|\frac{x^{n+1}}{n+2}\cdot\frac{n+1}{x^n}\right|=\lim_{n\to\infty}\left|\frac{n+1}{n+2}\right||x|=1\cdot|x|$

By the ratio test, the series will converge if $1\cdot|x|<1,\ i.e.\ -1<x<1\ \ or\ \ (-1,1)$

Test endpoints:

$x=-1: \sum_1^{\infty} \frac{(-1)^n(-1)^n}{n+1} = \sum_1^{\infty} \frac{1}{n+1}$ *is divergent harmonic*

$x=1: \sum_1^{\infty} \frac{(-1)^n(1)^n}{n+1} = \sum_1^{\infty} \frac{(-1)^n}{n+1}$ *is convergent by AST*

Interval is $(-1,1]$

6. $\lim_{n\to\infty}\left|\frac{\sqrt{n+1}x^{n+1}}{\sqrt{n}x^n}\right| = \lim_{n\to\infty}\left|\sqrt{\frac{n+1}{n}}\right||x| = 1\cdot|x|$

By the ratio test, the series will converge if

$1\cdot|x|<1,\ i.e.\ -1<x<1\ \ or\ (-1,1)$

Test endpoints:

$x=-1: \sum_1^{\infty}(-1)^n\sqrt{n}$ *is divergent nth term test.*

$x=1: \sum_1^{\infty}\sqrt{n}$ *is divergent.*

Interval is $(-1,1)$

8. $\lim_{n\to\infty}\left|\frac{(n+1)^{n+1}x^{n+1}}{n^n x^n}\right| = \lim_{n\to\infty}\left|(n+1)\left(\frac{n+1}{n}\right)^n\right||x| = \infty$

Converges only at center, $x=0$.

10.

$\lim_{n\to\infty}\left|\frac{10^{n+1}x^{n+1}}{(n+1)^3}\cdot\frac{n^3}{10^n x^n}\right| = \lim_{n\to\infty}\left|\frac{10n^3}{(n+1)^3}\right||x| = 10\cdot|x|$

By the ratio test, the series will converge if

$10\cdot|x|<1,\ i.e.\ \frac{-1}{10}<x<\frac{1}{10}\ \ or\ \left(\frac{-1}{10},\frac{1}{10}\right)$

Test endpoints:

$x=-1/10: \sum_1^{\infty}\frac{10^n(-1/10)^n}{n^3} = \sum_1^{\infty}\frac{(-1)^n}{n^3}$ *is abs.convergent p − series*

$x=1/10: \sum_1^{\infty}\frac{1}{n^3}$ *is convergent p − series.*

Interval is $\left(\frac{-1}{10},\frac{1}{10}\right)$

22. $\lim_{n\to\infty}\left|\frac{(n+1)(x-4)^{n+1}}{(n+1)^3+1}\cdot\frac{n^3+1}{n(x-4)^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(n^3+1)}{n((n+1)^3+1)}\right||x-4|$

$=1\cdot|x-4|$

By the ratio test, the series will converge if

$1\cdot|x-4|<1,\ i.e.\ -1<x-4<1\ \ or\ (3,5)$

Test endpoints:

$x=3: \sum_1^{\infty}\frac{n(-1)^n}{n^3+1}$ is absolutely convergent.

$x=5: \sum_1^{\infty}\frac{n(1)^n}{n^3+1}$ is convergent by comparison to $\sum_1^{\infty}\frac{1}{n^2}$

Interval is $[3,5]$

30. Since the center is x=0, the radius of convergence must be $4\le R<6$, therefore for a) x=1, is within the interval of convergence, so the series is convergent; b) x=8, is outside the interval of convergence, so divergent.; c) x=-3 is within the interval of convergence, thus convergent; d) x=-9 is outside interval of convergence, thus divergent.

C.

1. -1, converges

2. $\frac{1}{4}\arcsin\left(x^4\right)+C$

3. $4-\pi$

4a. $\frac{1}{2}\arctan(x)-\frac{x}{2\left(1+x^2\right)}+C$

4b. Same as 4a.

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ET Section 11.9 Representations of Functions as Power Series

A. EWA Homework 20 (optional)

B. ET Page 733 1, 3, 5, 7, 9, 15, 17, 23, 25, 27 – 30 all

C. Additional Problems

1. a) Use the geometric series to find a power series representation for $\frac{1}{1+x^2}$.

b) Use part (a) and integration to find a power series representation for arctan *x*, and find the interval of convergence.

c) Approximate $\int_0^{1/2} \arctan x \, dx$ by a power series.

d) What is an upper bound on the error if the sum of the first five terms of the series in (c) is used to approximate the values of the integral? (Hint: Use the Alternating Series Estimation Theorem.)

2. a) Previously, we found the series for $f(x)=\dfrac{1}{1+x}$ to be:

$\dfrac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n x^n$ and its interval of convergence to be $(-1,1)$. Use this series to find a power series for $\ln(1+x)$. Explain what process you will use to do this. Write the answer in summation notation.

b) Although the radius of convergence of this series is still 1, you need to test the endpoints (that is, $x=1,\ x=-1$) to see if they are included in the interval of convergence.

c) Use the series in (a) to approximate $\ln(1.2)$ with 3 decimal place accuracy.

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Answers:

B.

28.

$$\ln(1+x^4)=\int\frac{4x^3}{1+x^4}dx=4x^3\cdot\int\sum_{n=0}^{\infty}(-x^4)^n dx=4\int\sum_{n=0}^{\infty}(-1)^n x^{4n+3}dx$$

$$\ln(1+x^4)=4\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+4}}{4n+4}=\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+4}}{n+1}$$

$$\int_0^{0.4}\ln(1+x^4)dx=\int_0^{0.4}\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+4}}{n+1}dx=\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+5}}{(n+1)(4n+5)}\Bigg|_0^{0.4}=$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n 0.4^{4n+5}}{(n+1)(4n+5)}\Bigg|\approx 0.0020625=S_1$$

with $|error|<b_2\approx 1.7\cdot 10^{-7}$

30.

$$\int_0^{0.3}\frac{x^2}{1+x^4}dx=\int_0^{0.3}x^2\cdot\frac{1}{1-(-x^4)}dx=\int_0^{0.3}x^2\cdot\sum_{n=0}^{\infty}(-x^4)^n dx=$$

$$\int_0^{0.3}\sum_{n=0}^{\infty}(-1)^n x^{4n+2}dx=\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+3}}{4n+3}\Bigg|_0^{0.3}=\sum_{n=0}^{\infty}\frac{(-1)^n 0.3^{4n+3}}{4n+3}\approx$$

$S_1\approx 0.008968$ with $|\text{error}|<b_2=1.6\cdot 10^{-7}$

C.

1. **a)** $\dfrac{1}{1+x^2}=1-x^2+x^4-x^6+\ldots=\sum_{n=0}^{\infty}(-1)^n x^{2n}$

$-1<x<1$

b) arctan *x* =

$$\int\frac{1}{1+x^2}dx=\int\left(1-x^2+x^4-x^6+\ldots\right)dx$$

$$=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}+\ldots$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}\quad -1\le x\le 1$$

c) $\int_0^{1/2}\arctan x\,dx$

$$=\int_0^{1/2}\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}+..\right)dx$$

$$=\left(\frac{x^2}{2}-\frac{x^4}{4\cdot 3}+\frac{x^6}{6\cdot 5}-\frac{x^8}{8\cdot 7}+\frac{x^{10}}{10\cdot 9}-\frac{x^{12}}{12\cdot 11}+\ldots\right)\Bigg|_0^{1/2}$$

$$=\frac{1}{2^2\cdot 2}-\frac{1}{2^4\cdot 4\cdot 3}+\frac{1}{2^6\cdot 6\cdot 5}-\frac{1}{2^8\cdot 8\cdot 7}+\ldots$$

= $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n+2}(2n+2)(2n+1)}$

d) If the first five terms of the series are used to approximate the value of the integral then the | error | is less than $\frac{1}{2^{12} \cdot 12 \cdot 11}$ or 0.00000184955. This means that the sum is accurate to five decimal places.

2. a) Integrate the series $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$ because $\ln(1+x) = \int \frac{dx}{1+x}$. The integral of terms is $\int (1 - x + x^2 - x^3 + x^4 - \ldots)\, dx =$

$$C + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$$

b) $I = (-1, 1]$

c) $\ln(1+x) = \ln(1+.2)$, that is, let x=0.2 in the series for $\ln(1+x)$:

$$\ln(1.2) = (.2) - \frac{(.2)^2}{2} + \frac{(.2)^3}{3} - \frac{(.2)^4}{4} + \frac{(.2)^5}{5} - \ldots$$

Because $\frac{(.2)^5}{5} = 0.000064$ we will obtain ln(1.2) with error <.0005 if we add up the first 4 terms of the series: $\sum_{n=0}^{3} \frac{(-1)^n (0.2)^{n+1}}{n+1}$ =0.1822667 or 0.182 to 3 decimal places.

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ET Section 11.10 Taylor and Maclaurin Series

A. EWA Homework 16 (optional)

B. ET Page 746 5, 6, 7, 9, 13, 15, 17, 21, 29, 33, 37, 39, 41, 43, 51, 53, 54.

C. Additional Problems

For problems 1 – 2, write out 5 terms of the series that represents the integrand, integrate term by term and also write the answer to the integral in summation notation (note: use the Maclaurin series already found for $\arctan(x)$ and $\cos(x)$ to get started):

1. $\int \arctan(x^2)\, dx$

2. $\int \cos(e^x)\, dx$

3. Use the Maclaurin series for the $\cos(x)$ to approximate cos(10º) to four decimal places. (Convert to radians and use the Alternating Series Estimation Theorem.)

4. Use the Maclaurin series for e^x to approximate $e^{-0.2}$ with error less than 0.0005.

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B.

6. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$ for(-1,1)

54. $\int_0^{0.5} x^2 \cdot e^{-x^2} dx = \int_0^{0.5} x^2 \cdot \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} dx = \int_0^{0.5} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{n!} dx =$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{n!(2n+3)}\Bigg|_0^{0.5} = \sum_{n=0}^{\infty} \frac{(-1)^n (0.5)^{2n+3}}{n!(2n+3)}$$

To obtain $|\text{error}|$ <0.001, the error term is $b_2 \approx 0.00055$.

Therefor the approximating sum is $S_1 = 0.0354$

C.

1. $\arctan(x) = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$; substitute x^2 for *x* to obtain the series for $\arctan(x^2)$ $\arctan(x^2) = x^2-\frac{x^6}{3}+\frac{x^{10}}{5}-\frac{x^{14}}{7}+\frac{x^{18}}{9}-\frac{x^{22}}{11}+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+2}}{2n+1}$; now integrate this series term-by-term and write answer in summation notation:

$\int \arctan(x^2)dx = \frac{x^3}{3}-\frac{x^7}{7(3)}+\frac{x^{11}}{11(5)}-\frac{x^{15}}{15(7)}+\frac{x^{19}}{19(9)}-\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n x^{4n+3}}{(4n+3)(2n+1)}$.

2. $\cos(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$; substitute e^x for *x* to obtain the series for $\cos(e^x)$:

$\cos(e^x)=1-\frac{e^{2x}}{2!}+\frac{e^{4x}}{4!}-\frac{e^{6x}}{6!}+\frac{e^{8x}}{8!}-\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n e^{2n}}{(2n)!}$; now integrate this series term-by-term and write answer in summation notation: $\int \cos(e^x)dx = x-\frac{e^{2x}}{2\cdot 2!}+\frac{e^{4x}}{4\cdot 4!}-\frac{e^{6x}}{6\cdot 6!}+\frac{e^{8x}}{8\cdot 8!}-\ldots=C+x+\sum_{n=1}^{\infty}\frac{(-1)^n e^{(2n)x}}{n(2n)!}$

3. 10º= $\frac{\pi}{18}$ radians $\cos(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\frac{x^8}{8!}-\ldots$ and $\cos\left(\frac{\pi}{18}\right)=1-\frac{\left(\frac{\pi}{18}\right)^2}{2!}+\frac{\left(\frac{\pi}{18}\right)^4}{4!}-\ldots$ Because $\frac{\left(\frac{\pi}{18}\right)^4}{4!}\approx 0.0000386$ which is less than 0.00005, to obtain four decimal place accuracy, add the first two terms of the series for $\cos\left(\frac{\pi}{18}\right) =1-\frac{\left(\frac{\pi}{18}\right)^2}{2!}\approx 0.984769$ or 0.9848 with four place decimal accuracy.

4. $e^{-x}=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$. Therefore, $e^{-0.2}=1-(0.2)+\frac{(0.2)^2}{2!}-\frac{(0.2)^3}{3!}+\frac{(0.2)^4}{4!}-\ldots$ $\frac{(0.2)^4}{4!}\approx 0.0000666<0.0005$ so to obtain $e^{-0.2}$ accurate to 3 decimal places, add the first 4 terms of the series: $\sum_{n=0}^{3}\frac{(-1)^n (0.2)^n}{n!}\approx 0.818666$ or 0.819.

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ET Section 11.11 Applications of Taylor Polynomials

A. ET Page 755 1, 3, 5, 13, 19, 23, 25, 27, 31

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