Trigonometry Review for Calculus Students

The trigonometric skills that you will need to possess for the three courses in Brookdale's Calculus sequence (171, 172, 273) are presented below. Much of the material comes from your textbook's Appendix C, pp. A18 – A27. In the pages that follow, whenever you come across a statement like "*you need to know*," understand that it means you are expected to have the information memorized.

(I) Angles

One definition of calculus is "the study of one and two-variable functions over the Real Number system." Since angles measured in degrees are not part of the real number system, when solving angular problems in calculus, angles will be expressed in radian mode. **You must know the 17 special angles in their radian form.** A table of these values is presented on page A18 of your text's appendix. They must be memorized.

(II) The Graphs of the six trig functions

It is necessary for you to be able to draw a quick sketch of each of the six trig functions with the appropriate angles labeled. These are supplied in Figure 13 (p. A24) and Figure 14 (p. A25) of your text's appendix. Memorize them. You also need to know the period of each of the six trig functions. This is easy: all of the trig functions have a period of 2π except for tangent and cotangent each of which have a period of π.

(III) Right-Triangle Trig

You should know, from memory, Table 4 on page A20 in your text. A mnemonic for recalling this is SOHCAHTOA (Sine = Opposite over Hypotenuse, etc.) In addition to that, you must also know the reciprocal functions of the sine, cosine and tangent. These are, respectively,

$$\csc(\theta) = \frac{1}{\sin(\theta)} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} \qquad \cot(\theta) = \frac{1}{\tan(\theta)}.$$

(IV) Other Identities

The following, from pages A22-A23, must be memorized.

For MATH 171: 7, 8, 9, 15a, 15b

For MATH 172: all the above plus 16a, 16b, 17a, 17b

Practice Set 1 (Answers follow)

Match the expressions in Column A with the equivalent expressions in Column B.

Column A	Column B
a. $1+\cot^2\theta$	**i.** $\sec^2\theta$
b. $\sec\theta$	**ii.** $2\sin\theta\cos\theta$
c. $\tan\theta$	**iii.** $\csc^2\theta$
d. $\sin^2\theta+\cos^2\theta$	**iv.** $\dfrac{1}{\cos\theta}$
e. $\tan^2\theta+1$	**v.** $\dfrac{\sin\theta}{\cos\theta}$
f. $\sin 2\theta$	**vi.** 1

Answers to Practice Set 1

a. (iii) **b.** (iv) **c.**(v) **d.**(vi) **e.**(i) **f.**(ii)

(V) Evaluating Trig Expressions

To **evaluate a trig function** means to find the value of the trig function at a particular angle. If you evaluate $\cos\left(\frac{11\pi}{6}\right)$ you get the number $\frac{\sqrt{3}}{2}$. So, what is $\sin\left(\frac{7\pi}{6}\right)$? How about $\sec\left(\frac{5\pi}{3}\right)$? And what is the value of $\tan\left(\frac{3\pi}{4}\right)$?

Evaluating trig functions is a **critical** skill that is used in each of the three big areas of calculus: limits & continuity, differentiation, and integration.

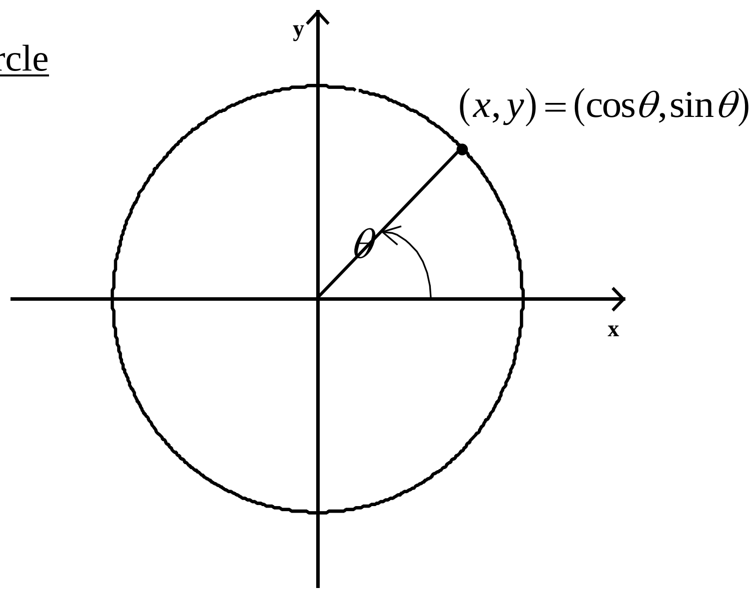
You must be able to present the trigonometric value of any of the 17 special angles mentioned in (I) above.

There are two ways to do these evaluations.

Method 1: Memorize the diagram of the Unit Circle which is on the next page.

How to Use the Unit Circle

By definition, in the Unit Circle the *x*-coordinate is defined as the cosine of angle and the *y*-coordinate is defined as the sine of the angle.



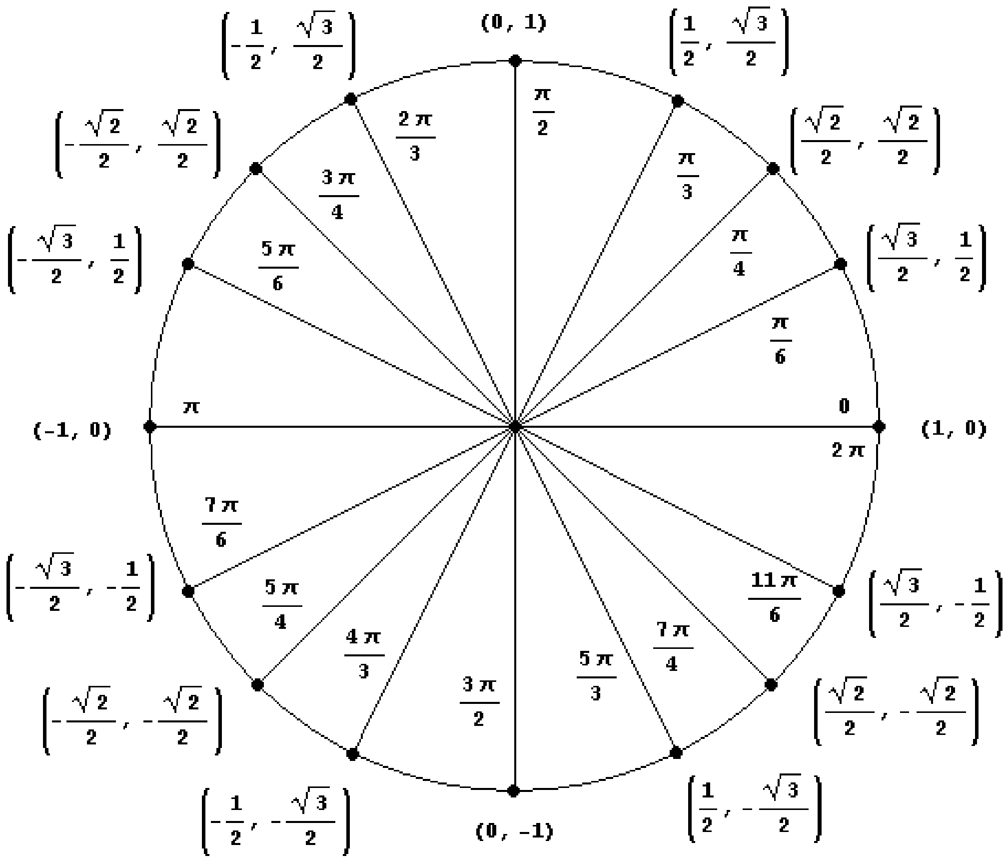
Using the diagram on the next page, you can see that $\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

(which is the value of the *y*-coordinate associated with the angle of $\frac{4\pi}{3}$).

In addition, $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (the *x*-coordinate associated with the angle $\frac{7\pi}{4}$).

And note that $\sin\left(\frac{\pi}{2}\right) = 1$ whereas $\cos\left(\frac{\pi}{2}\right) = 0$ (the *y* and *x* coordinates associated with the angle $\frac{\pi}{2}$).

The Unit Circle



The diagram above only gives you the values of cosine (x) and sine (y) for the 17 special angles. To find the values of the four reciprocal trig functions (tangent, cotangent, secant, and cosecant), use the definitions in Box 6 at the top of p. A22. Two examples follow.

Example 1:

$$\tan\left(\frac{7\pi}{6}\right) = \frac{\sin(7\pi/6)}{\cos(7\pi/6)} = \frac{(-1/2)}{\left(-\sqrt{3}/2\right)} = \frac{1}{\sqrt{3}}$$

Example 2:

$$\sec\left(\frac{\pi}{2}\right) = \frac{1}{\cos(\pi/2)} = \frac{1}{0} = \text{undefined}$$

Method 2: Use the two special right-triangles and C.A.S.T. This method requires a bit more work than the Unit Circle but it involves much less memorization. After reviewing both methods, choose the one with which you are more comfortable.

First you must have memorized the two special right-triangles shown in Figure 8 on page A20 of your text.

Next you must be able to determine the **reference** **angle** of a given angle. In radian mode this is very easy when you are working with any of the 17 special angles. If the angle falls in $Q_I \left(0, \frac{\pi}{2}\right)$, the angle itself is a reference angle (by definition, a *reference* *angle* is any angle between 0 and $\frac{\pi}{2}$). Thus the reference angle for $\frac{\pi}{6}$ *is* $\frac{\pi}{6}$ since the angle falls within $\left(0, \frac{\pi}{2}\right)$. If the angle is greater than $\frac{\pi}{2}$, then you get the reference angle by taking π and placing it over the denominator of the non-reference angle you are given. The reference angle for $\frac{5\pi}{4}$, which is not within $\left(0, \frac{\pi}{2}\right)$, will be $\frac{\pi}{4}$ since the original angle had a denominator of 4. It's that simple!

Practice Set 2

Find the reference angle for: a) $\frac{5\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{3\pi}{4}$ d) $\frac{4\pi}{3}$

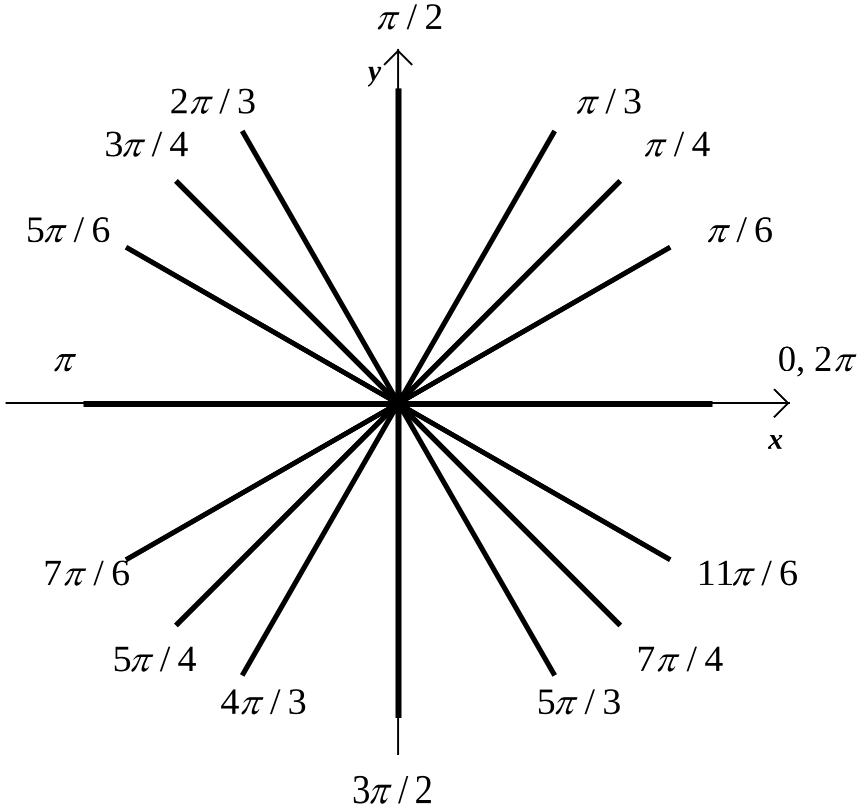
Answers to Practice Set 2

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

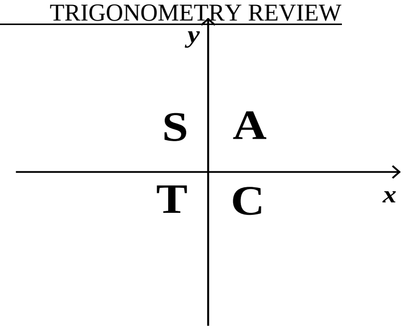
Next, you must know in which quadrant each of the 17 special angles falls. Below is a summary that must be memorized. If you are more visually oriented, the graph below can be memorized instead.

Quadrant I: $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ Quadrant II: $\frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}$

Quadrant III: $\frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}$ Quadrant IV: $\frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}$



Finally, to use this method you must also know when the six trig functions are positive and when they are negative. For example: is $\sin\left(\frac{11\pi}{6}\right)$ positive or negative? Is $\tan\left(\frac{5\pi}{4}\right)$ positive or negative? How do you determine the sign of each of the trig functions when evaluated at specific angles?



The easiest way to do this is to memorize C.A.S.T. Starting in Quadrant I, the four letters stand for **A**ll, **S**ine, **T**angent, and **C**osine. A mnemonic for this is "**A**ll **S**tudents **T**ake **C**alculus." These letters dictate which trig functions are **positive** in each of the four quadrants. So, in Q_I $\left(0, \frac{\pi}{2}\right)$, all six trig functions are positive. In Q_{II} $\left(\frac{\pi}{2}, \pi\right)$, only the sine function (and its reciprocal, cosecant) are positive; the other four trig functions are negative if they are evaluated at an angle that falls within $\left(\frac{\pi}{2}, \pi\right)$. In Q_{III} $\left(\pi, \frac{3\pi}{2}\right)$, only the tangent function and its reciprocal, cotangent, are positive. And in Q_{IV} $\left(\frac{3\pi}{2}, 2\pi\right)$, only the cosine function and its reciprocal (secant) are positive.

Practice Set 3

Using CAST, determine the sign (positive or negative) of each of the expressions below.

a) $\sin\left(\frac{7\pi}{6}\right)$ b) $\tan\left(\frac{4\pi}{3}\right)$ c) $\sec\left(\frac{5\pi}{4}\right)$

d) $\csc\left(\frac{\pi}{3}\right)$ e) $\cos\left(\frac{7\pi}{4}\right)$ f) $\sin\left(\frac{2\pi}{3}\right)$

Answers to Practice Set 3

a) negative b) positive c) negative d) positive e) positive f) positive

You are now ready to evaluate any of the six trig functions at 13 of the 17 special angles (the remaining four special angles, known as quadrantal angles, will be discussed later). This evaluation is a three-step process.

Step 1: Using the given angle's denominator, get the reference angle. Then, based on the trig function and the quadrant in which the original angle falls, determine if the value will be positive or negative.

Step 2: Using SOHCAHTOA and the two right triangles in Fig 8, determine the value of the trig expression.

Step 3: If necessary, adjust the expression to reflect the appropriate quadrant by placing a negative sign in front.

Example 1

What is $\cos\left(\frac{7\pi}{6}\right)$?

Step 1: The reference angle is $\frac{\pi}{6}$ and since $\frac{7\pi}{6} \in Q_{III}$ where cosine < 0

$\cos\left(\frac{7\pi}{6}\right)$ will be negative. (A negative sign will be added in Step 3.)

Step 2: Using the CAH part of SOHCAHTOA and the 2nd right triangle in Fig 8, you determine that $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

Step 3: Since we noted in Step 1 that $\cos\left(\frac{7\pi}{6}\right)$ must be negative, we add a negative sign in front of $\frac{\sqrt{3}}{2}$. Thus $\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$.

Example 2

What is $\sin\left(\frac{3\pi}{4}\right)$?

Step 1: Reference angle = $\frac{\pi}{4}$. As $\frac{3\pi}{4}$ falls in Q_{II} where sine > 0 ,

$\sin\left(\frac{3\pi}{4}\right)$ will be positive.

Step 2: Using the right triangle that has $\frac{\pi}{4}$ and your knowledge of SOHCAHTOA, you can now evaluate $\sin\left(\frac{3\pi}{4}\right)$. Using the 1st triangle in Fig 8, and the definition that $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$, you get $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

Step 3: Since no adjustment in sign is necessary, $\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

Example 3

What is $\tan\left(\frac{5\pi}{3}\right)$?

Step 1: Reference angle = $\frac{\pi}{3}$. Also $\frac{5\pi}{3} \in Q_{IV}$ and tangent < 0 there so $\tan\left(\frac{5\pi}{3}\right)$ will be negative. (A negative sign will be added in Step 3.)

Step 2: Using the TOA part of SOHCAHTOA and the 2nd right triangle in Fig 8, you determine that $\tan\left(\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{1}$.

Step 3: In Step 1 we noted that $\tan\left(\frac{5\pi}{3}\right)$ must be negative, so now we add a negative sign in front of $\frac{\sqrt{3}}{1}$. We now have: $\tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$.

What about trigonometric values of angles that don't fall within one of the two triangles? These angles, called **quadrantal angles**, are multiples of $\frac{\pi}{2}\left[\text{examples: } 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \textit{etc.}\right]$.

The most direct way of evaluating a trigonometric function at a quadrantal angle is to refer to the graph of the trig function in question. Once you have memorized the six graphs on pp. A24-25 of your textbook's Appendix, you should try the problems in Practice Set 4.

Practice Set 4

Find the value of: a) $\sin\left(\frac{3\pi}{2}\right)$ b) $\tan(\pi)$ c) $\cos(0)$ d) $\sec\left(\frac{\pi}{2}\right)$

e) $\csc\left(\frac{\pi}{2}\right)$ f) $\sin(\pi)$ g) $\cot(0)$ h) $\cos\left(\frac{\pi}{2}\right)$

Answers to Practice Set 4

a) -1 b) 0 c) 1 d) undefined e) 1 f) 0 g) undefined h) 0

Note

If you are pressed for time there is an alternate method for evaluating the tangent, cotangent, secant and cosecant functions at quadrantal angles. This method uses the identities in Box 6 at the top of page A22 along with the graphs of the sine and cosine function. Here are three examples.

$$\tan(\pi) = \frac{\sin(\pi)}{\cos(\pi)} = \frac{0}{-1} = 0$$

$$\sec\left(\frac{3\pi}{2}\right) = \frac{1}{\cos(3\pi/2)} = \frac{1}{0} = \text{undefined}$$

$$\csc\left(\frac{\pi}{2}\right) = \frac{1}{\sin(\pi/2)} = \frac{1}{1} = 1.$$

Putting It All Together

Once you have studied Method 2 (or have memorized the Unit Circle which is Method 1), do the problems in Practice Set 5 on the next page *without* looking at your notes. That is the only way you will know if you can do these problems.

Practice Set 5

Find the value of each of the following expressions.

a) $\sin\left(\frac{7\pi}{6}\right)$ b) $\sec\left(\frac{5\pi}{3}\right)$ c) $\tan\left(\frac{3\pi}{4}\right)$ d) $\cos(\pi)$ e) $\tan\left(\frac{3\pi}{2}\right)$

f) $\cos\left(\frac{5\pi}{6}\right)$ g) $\cot\left(\frac{5\pi}{3}\right)$ h) $\sin\left(\frac{\pi}{2}\right)$ i) $\sin(0)$ j) $\sec(\pi)$

k) $\tan\left(\frac{7\pi}{6}\right)$ l) $\cos\left(\frac{5\pi}{4}\right)$ m) $\csc(\pi)$ n) $\sin\left(\frac{5\pi}{3}\right)$ o) $\sec\left(\frac{\pi}{4}\right)$

Answers to Practice Set 5

a) $-\frac{1}{2}$ b) 2 c) -1 d) -1 e) undefined f) $-\frac{\sqrt{3}}{2}$

g) $-\frac{1}{\sqrt{3}}$ h) 1 i) 0 j) -1 k) $\frac{1}{\sqrt{3}}$ l) $-\frac{1}{\sqrt{2}}$

m) undefined n) $-\frac{\sqrt{3}}{2}$ o) $\sqrt{2}$

(VI) Solving Trig Equations

Using three examples, we will review the method for solving trig equations. Then you will have number of trig equations to solve on your own. At this point, you are expected to have memorized all the previous material in this review packet.

Example 1 Solve $2\sin(x)-1=0$ for all solutions.

First, isolate the trig expression and make sure it has a coefficient of 1.

$$\begin{aligned} 2\sin(x) - 1 &= 0 \\ 2\sin(x) &= 1 \\ \sin(x) &= \frac{1}{2} \end{aligned}$$

This equation requires that you find the angles (x) which produce a sine value of ½. The first step in solving trig equations is to determine the reference angle. This amounts to answering the question: within $\left(0, \frac{\pi}{2}\right)$ what angle x generates a sine value of ½? Using either Method 1 or Method 2 recall that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ so the reference angle $x = \frac{\pi}{6}$. Now you must determine in which quadrants the answer lies. Because ½ > 0 you must determine in which two quadrants sin (x) > 0. Using C.A.S.T, you determine those quadrants are I and II. (You will <u>always</u> come up with two quadrants initially. Sometimes, as in Example 2, the final answer will have only a single quadrant.)

For Q_I, we get $x = \frac{\pi}{6}$. For Q_{II}, we get $x = \frac{5\pi}{6}$. This second answer is found by subtracting the reference angle $\left(\frac{\pi}{6}\right)$ from π: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

$\frac{\pi}{6}$ and $\frac{5\pi}{6}$ are not, however, the final answers.

Remember that the period of the sine curve is 2π; thus it repeats every 2π. The full answer must include this information in the solution. To do that we add an "extension" onto the two angles we found $\left(\frac{\pi}{6} \text{ and } \frac{5\pi}{6}\right)$. This extension represents the periodic nature of the answer. Since the sine curve's period is 2π, the extension is expressed as $2n\pi$ (where n represents any integer).

The final answer:

$$Q_I: \quad x = \frac{\pi}{6} + 2n\pi$$

$$Q_{II}: \quad x = \frac{5\pi}{6} + 2n\pi$$

Example 2 Solve $\sqrt{3}+\tan(\theta)=0$ for all solutions.

Isolating the trig function you have: $\tan(\theta)=-\sqrt{3}$. First get the reference angle. Using either Method 1 or Method 2, you recall that the angle θ which produces a tangent value of $\sqrt{3}$ is $\frac{\pi}{3}$. Since the numeric value ($-\sqrt{3}$) is negative, you must determine in which quadrants is $\tan(\theta)<0$? Using C.A.S.T., the two quadrants are Q_{II} & Q_{IV}. You now place the reference angle in the appropriate two quadrants and determine the answers.

For Q_{II} you subtract the reference angle from pi: $\pi-\frac{\pi}{3}=\frac{2\pi}{3}$. The final answer includes the extension. The extension is often the trig function's period with an *n* added on. For tangent, the period is π so the final answer is $\theta=\frac{2\pi}{3}+n\pi$.

For Q_{IV}, you subtract the reference angle from 2π: $2\pi-\frac{\pi}{3}=\frac{5\pi}{3}$ and then add on the extension: $\theta=\frac{5\pi}{3}+n\pi$.

Finally, for functions in which the period is π, the two answers can often be "compressed" into a single answer (notice that the distance between $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$ is π). So, the final solution to this equation is: $\theta=\frac{2\pi}{3}+n\pi$. (If you add π to $\frac{2\pi}{3}$ you get the second answer, the one in Q_{IV}: $\frac{5\pi}{3}$, so there is no reason to list two answers once you've compressed the answer into a single solution.)

Example 3 Find all solutions to $\sqrt{2}\cos(t)=-1$.

Rewrite the equation so the trig expression's coefficient is 1: $\cos(t)=-\frac{1}{\sqrt{2}}$. What is the reference angle for this equation? What is the angle at which the cosine function generates the value of $\frac{1}{\sqrt{2}}$? Using either

Method 1 or 2 you know that the reference angle is $\frac{\pi}{4}$. Now, in what two quadrants is cos (t) < 0? Answer: Q_{II} & Q_{III}. Thus your answers must fall in those two quadrants. To find the answer in Q_{II} subtract the reference angle from pi: $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$. To find the answer in Q_{III}, add the reference angle to pi: $\pi + \frac{\pi}{4} = \frac{5\pi}{4}$. Finally add the extension $2n\pi$ to both answers to get:

$$Q_{II}: \quad t = \frac{3\pi}{4} + 2n\pi$$

$$Q_{III}: \; t = \frac{5\pi}{4} + 2n\pi$$

Practice Set 6

Find all solutions to each equation.

a) $\tan(\theta) = 1$ b) $\sin(\theta) + 1 = 0$ c) $-2\cos(y) = 1$

d) $2\sin(x) + \sqrt{3} = 0$ e) $\sqrt{3} - 2\cos(x) = 0$ f) $3\tan(x) = 0$

g) $\cos(t) + 1 = 0$ h) $\sqrt{2}\sin(\theta) = 1$

Answers to Practice Set 6

a) $\theta = \frac{\pi}{4} + n\pi$ (This single answer is arrived by compressing the two answers of $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ into the single answer with an extension of $n\pi$)

b) $\theta = \frac{3\pi}{2} + 2n\pi$ c) $y = \frac{2\pi}{3} + 2n\pi, \quad y = \frac{4\pi}{3} + 2n\pi$

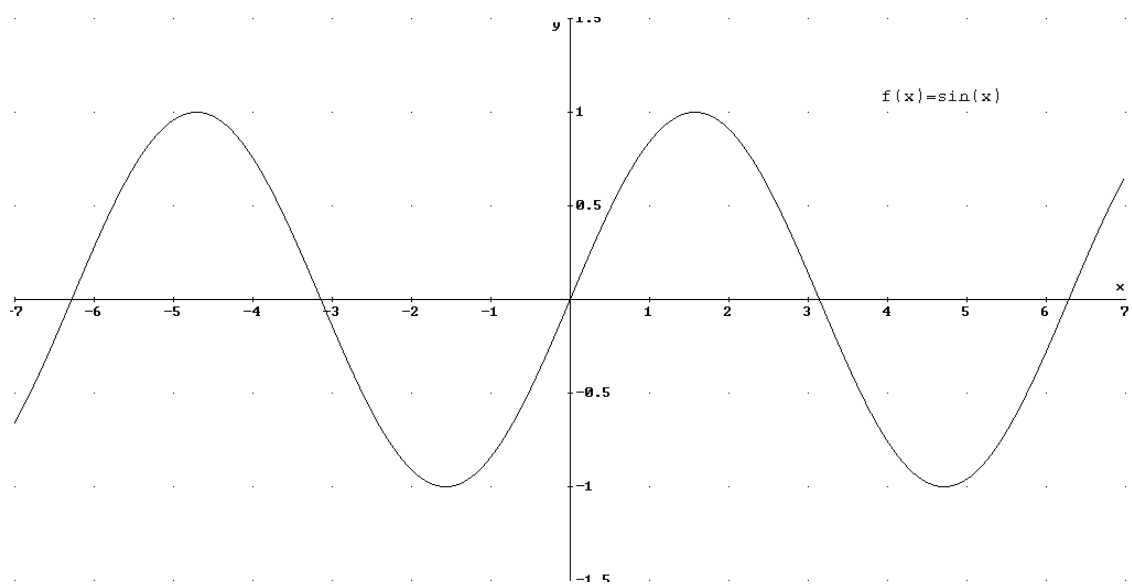
d) $x = \frac{4\pi}{3} + 2n\pi, \quad x = \frac{5\pi}{3} + 2n\pi$ e) $x = \frac{\pi}{6} + 2n\pi, \; x = \frac{11\pi}{6} + 2n\pi$

f) $x = n\pi$ g) $t = \pi + 2n\pi$ h) $\theta = \frac{\pi}{4} + 2n\pi, \quad \theta = \frac{3\pi}{4} + 2n\pi$

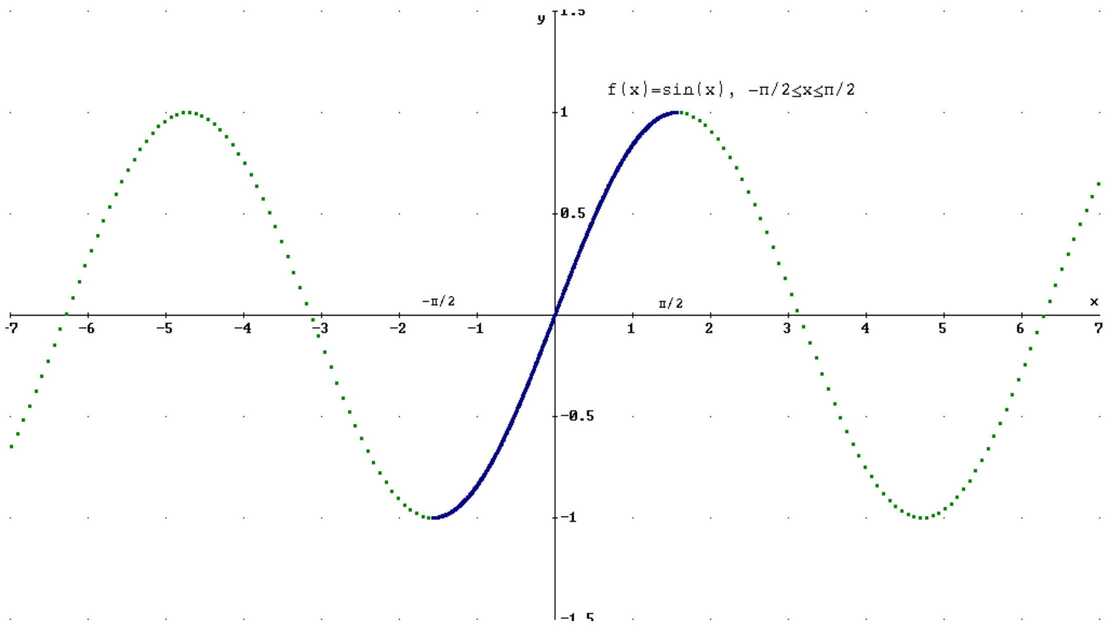
(VII) Inverse Trigonometric Functions

The six trigonometric functions are periodic, so they do not have inverses unless their domains are appropriately restricted. Each function has its domain restricted (making it one-to-one), so that an inverse function can be created. For the purposes of our calculus sequence you will need to know the inverse functions of the sine, cosine, tangent and secant. These are detailed in the material which follows.

1. Consider $f(x) = \sin(x),\ x \in \Re$. The graph shows that *f* is not a *one-to-one* function.

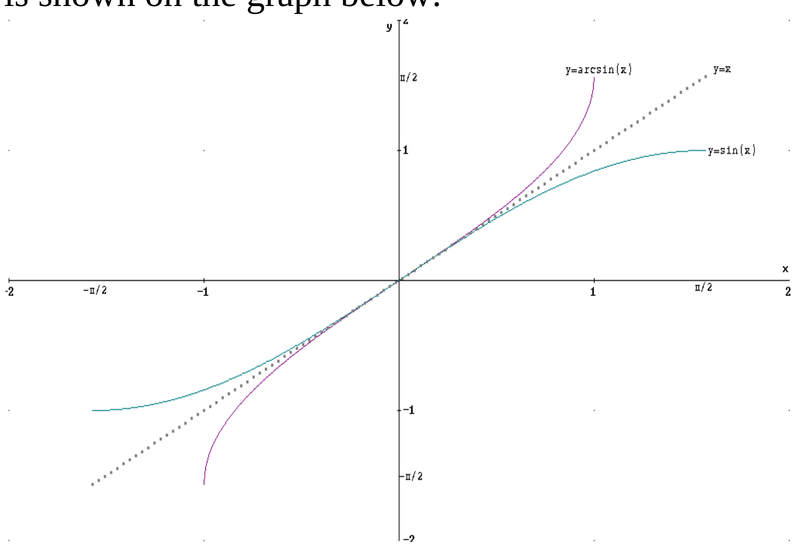


The obvious restriction on the domain that will make *f one-to-one* and still cover the entire range of the function is to let *x* vary from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$, i.e. $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. This is illustrated in the graph at the top of the next page:

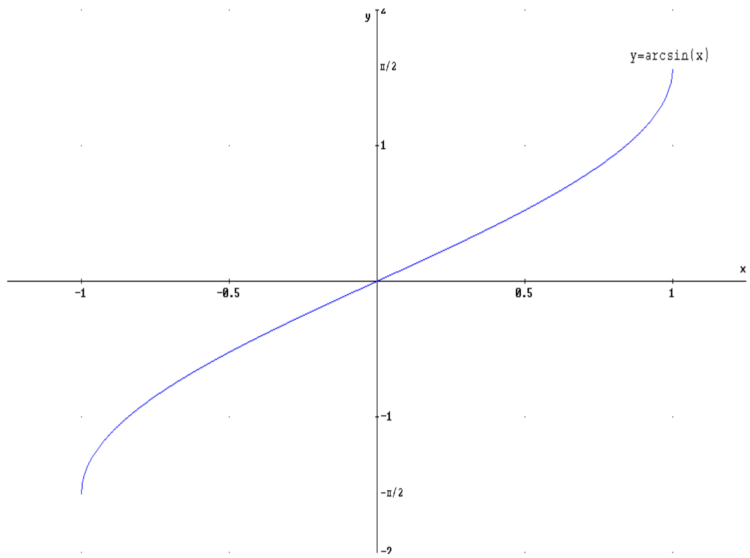


The inverse sine function is created by reversing the roles of the variables x and y. Let $f(x)=\sin(x),$ for $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ $\Leftrightarrow$ $y=\sin(x)\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Then f^{-1} is defined as the set of ordered pairs (x, y) such that $x=\sin(y),$ for $y\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$. We cannot solve for y algebraically, so we create a new notation for the inverse function. The inverse sine function is denoted by $f^{-1}(x)=\sin^{-1}(x)$ or $f^{-1}(x)=\arcsin(x)$. The graph of f^{-1} can be obtained by rotating the graph of $f(x)=\sin(x),$ for $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ about the line $y=x$. This is shown on the graph below:

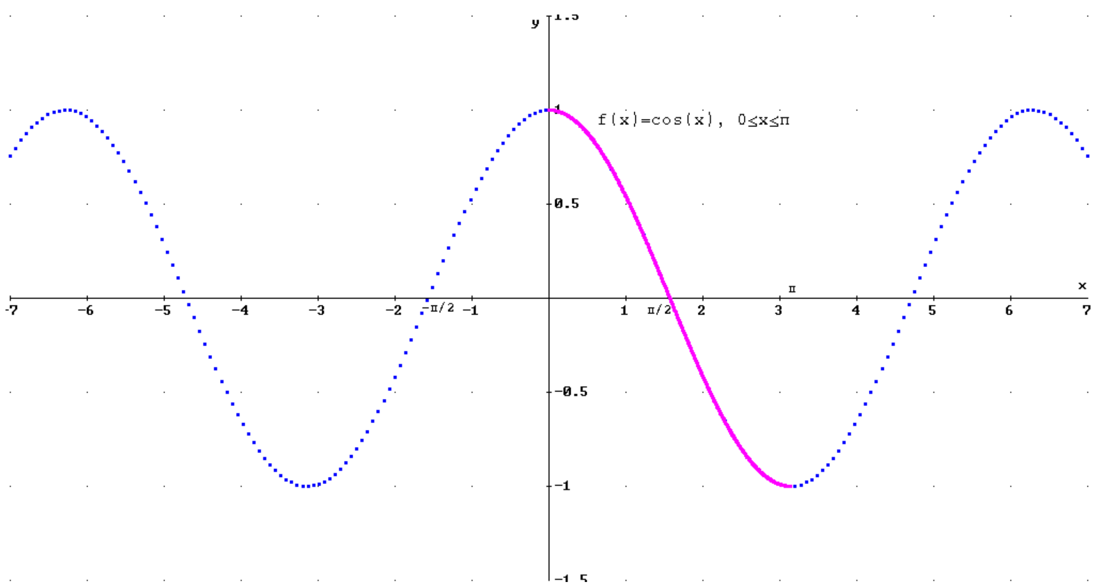


So the graph of the inverse sine function is:

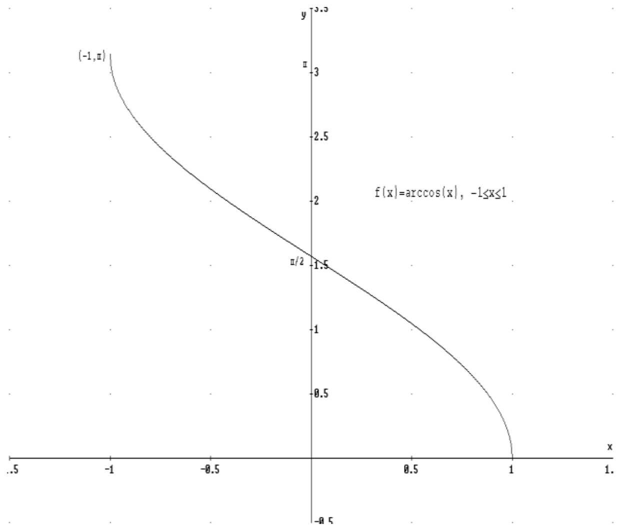


Domain: $-1 \le x \le 1$, Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

2. Now let $f(x) = \cos(x), \text{for } x \in [0, \pi]$. The graph is shown below. Note the solid part of the graph.

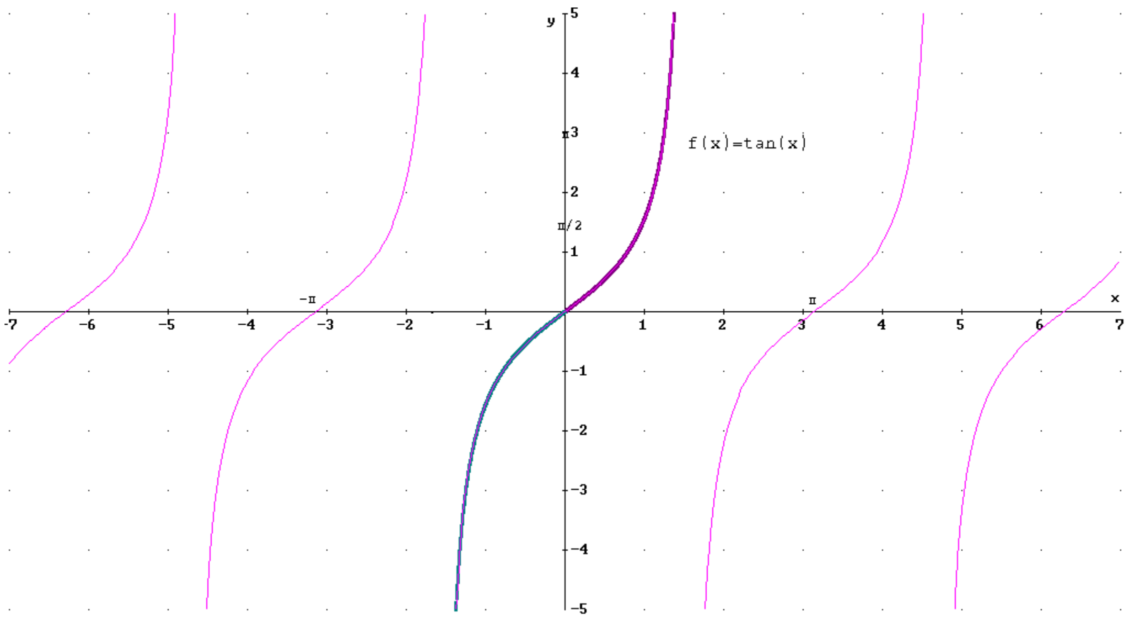


The restriction on the domain to make *f one-to-one* and still cover the entire range of the function is $x \in [0, \pi]$. The inverse cosine function is denoted by $f^{-1}(x) = \cos^{-1}(x)$, or $f^{-1}(x) = \arccos(x)$. The graph of f^{-1} is obtained by rotating the highlighted part of the cosine curve (above) around the line $y = x$. The graph of $f^{-1}(x) = \cos^{-1}(x)$ appears at the top of the next page:



Domain: $-1 \le x \le 1$, Range: $0 \le y \le \pi$

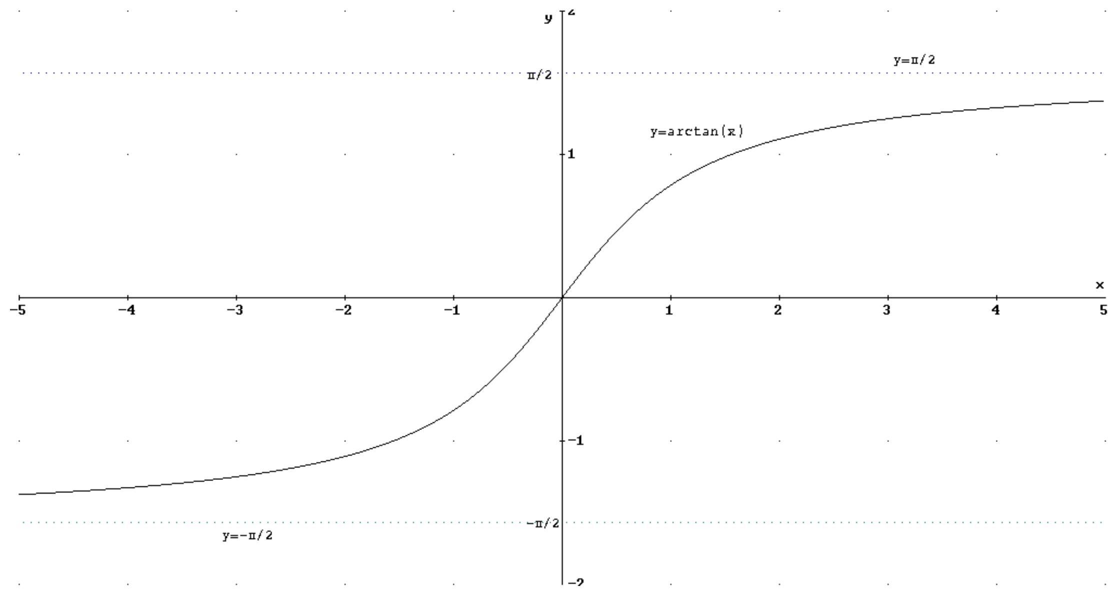
3. Let $f(x) = \tan(x)$, for $x \neq \frac{\pi}{2} + n\pi, n \in I$. The graph is shown below with the branch used to create the inverse function highlighted.:



The domain will be restricted to the *open* interval from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$:

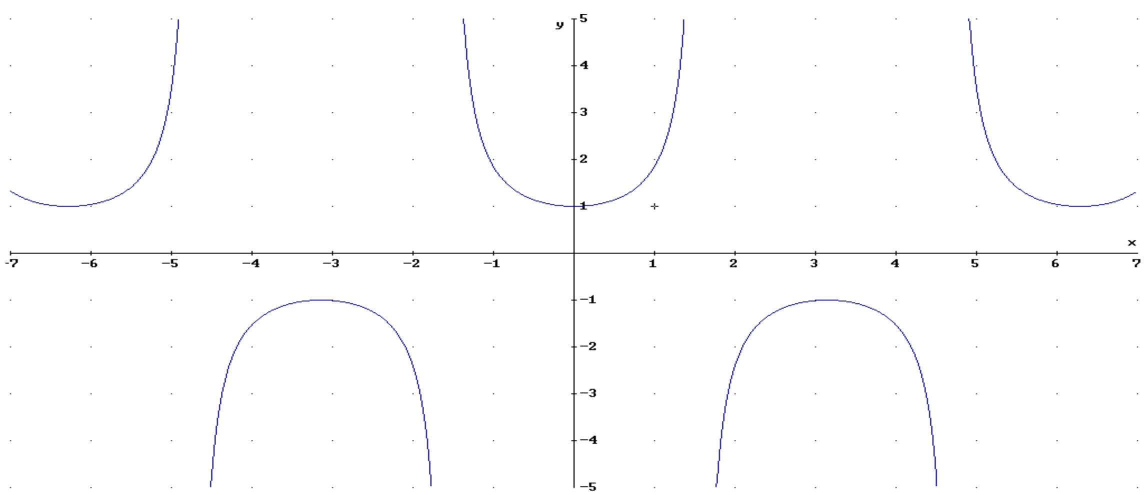
i.e., $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

The inverse tangent function is denoted by $f^{-1}(x) = \tan^{-1}(x) = \arctan(x)$. The graph of f^{-1} is:



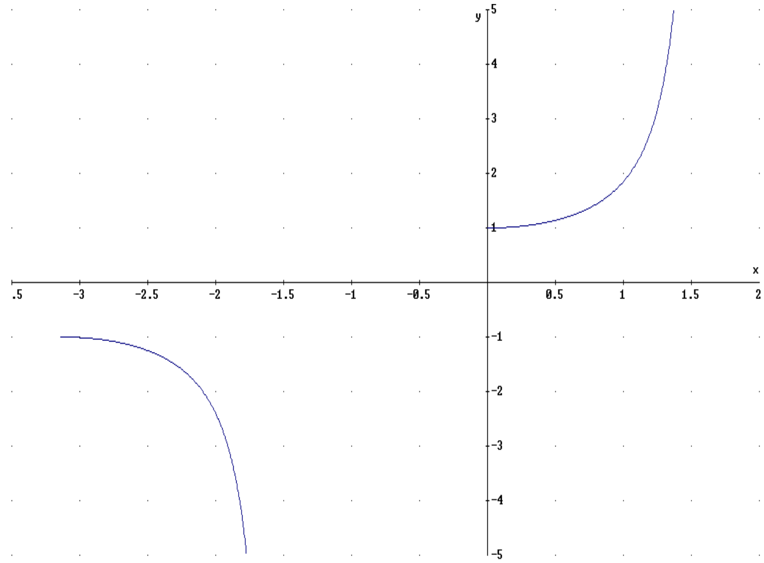
Domain: $(-\infty, \infty)$, Range: $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

4. Let $f(x) = \sec(x)$, for $x \neq \dfrac{\pi}{2} + n\pi,\ n \in I$. The graph is shown below:



The range of this function is the union of two disjoint intervals, i.e. $(-\infty, -1] \cup [1, \infty)$. For the *positive* part of the range we will use $x \in \left[0, \dfrac{\pi}{2}\right)$.

There is no general agreement on the restriction of the domain for the *negative* part of the range. The choices include $\left[-\pi,-\frac{\pi}{2}\right)$ or $\left(\frac{\pi}{2},\pi\right]$ or $\left[\pi,\frac{3\pi}{2}\right)$. Each choice has some advantages and some disadvantages when applied to calculus. For our purposes, we will create the inverse secant function by limiting the domain to $x \in \left(-\pi,-\frac{\pi}{2}\right] \cup \left[0,\frac{\pi}{2}\right)$. This produces:



In order to create the graph of the inverse function, arcsec (x), we reverse the domain and range. This produces the graph of $f^{-1}(x) = \sec^{-1}(x) = \operatorname{arcsec}(x)$: