Modeling Rocket Launch Physics

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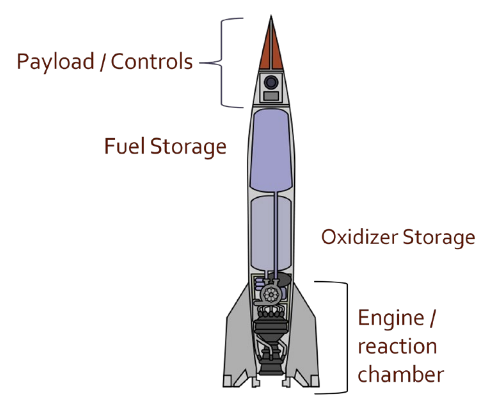
Objective

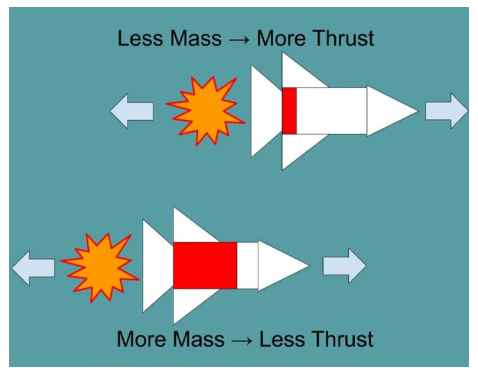
To implement a program that mathematically models elements of a rocket launch using Tsiolkovsky's Rocket Equation.

Rocket Basics

- Rockets combine a fuel and oxidizer to produce thrust to send a payload to space
- As the rocket burns, fuel is consumed making the rocket weigh less over time, causing rocket thrust to be more efficient/stronger



Rocket Fuel Dilemma



- Adding more propellant to the rocket means the rocket has more mass
- This slows the rocket down, and causes the rocket to require more propellant to achieve the same velocity

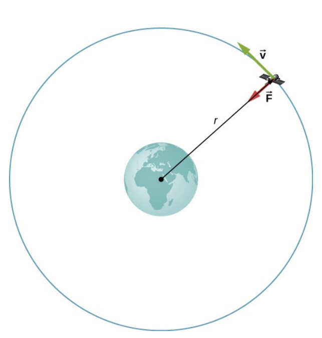
Tsiolkovsky's Rocket Equation

$$\Delta v = v_e \cdot \ln \frac{m_0}{m_f}$$

Δv : Maximum change in velocity of the rocket (m/s)
v_e : Effective exhaust velocity of the thrusters (m/s)
m_0: Mass of payload + mass of fuel (wet mass) (kg)
m_f: Mass of payload (dry mass) (kg)

Orbital Velocity

- To reach orbit, an object needs to move faster than Earth's gravitational pull
- Rockets only go straight up for a limited time, and then move sideways to reduce amount of fuel needed
- The orbital velocity is: $V_{orbit} = \sqrt{\frac{GM}{r}}$



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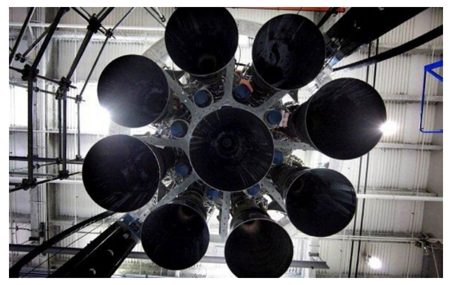
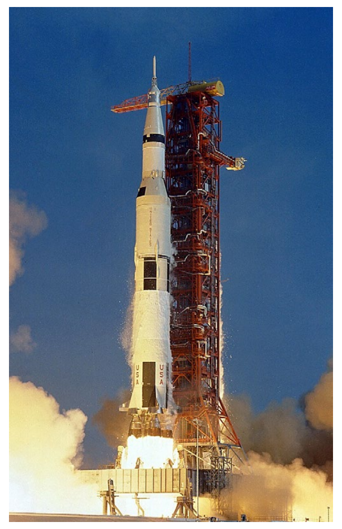
Modeling a SpaceX Falcon 9 V 1.2 Rocket Launch

- **SpaceX:** Private launch service provider for commercial, scientific and military payloads
- **Falcon 9:** Rocket designed by SpaceX to be reusable and retrieved after launch
- Over 300 successful Falcon 9 launches since 2010



Falcon 9 Stage 1 Engines

SpaceX's Falcon 9



Saturn V

	Falcon 9 Stage 1 / Stage 2	Saturn V Stage 1
Fuel / Oxidizer	RP-1 – LOX	RP-1 - LOX
Specific Impulse, Effective Exhaust velocity	283 s, 2,773.4	263 s, 2,577.4
	348 s (v), 3,410.4	
Dry Mass	27.2 tonnes	130 tonnes
	4.5 tonnes	
Wet Mass	483.2 tonnes	2,214 tonnes of fuel
	116.0 tonnes	
b_t/R	162 seconds / 2.537037 tonnes/s *	168 seconds / 12.40476 tonnes/s
	397 seconds / 0.289 tonnes/s *	

$R = burn\ Rate$ $\quad b_t = Burn\ Time$

$R = \frac{amount\ of\ fuel}{b_t}$ $\quad v_e = I_{sp} \cdot g_0$

$1\ tonne/metric\ ton = 1000kg$

- Obtained rocket characteristics from publicly available data sheets
- Implemented a program that calculates eleven different launch-related values, based on rocket attributes
- Evaluated accuracy with examples using real-world data from SpaceX and NASA to ensure values are reasonable
- Calculated orbital velocities for several planetary bodies at various altitudes

Planetary Body (Mass) (Radius)	ΔV required to reach Low Orbit (Altitude)	ΔV to reach GEO Orbit
Earth ($5.9 \cdot 10^{24}\ kg$) (6,378 km)	7 km/s (<2,000 km)	9 km/s (35,786 km)
Mars ($6.42 \cdot 10^{23}\ kg$) (3,396 km)	3 km/s (<2,000 km)	1.5 km/s (13,634 km)

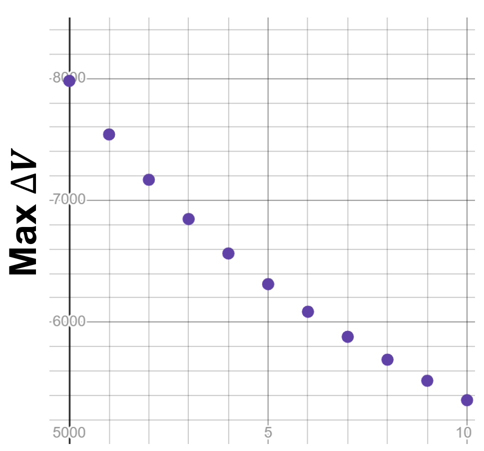
Implementation

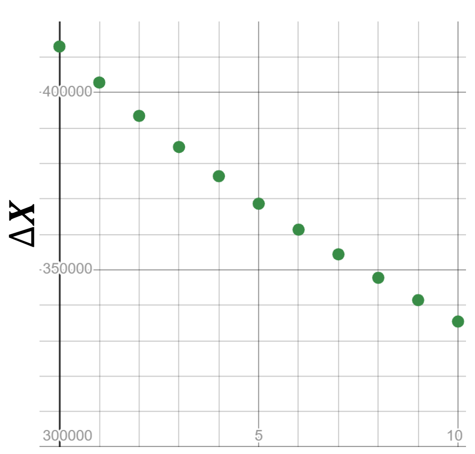
- Designed a set of Java functions, that may call each other, to compute and output launch-related values
- Allows for simple substitution of rocket characteristics, such as amount of fuel, type of engine, payload mass, burn time, etc., and repeated application with different characteristic values (e.g., payload mass)

Example Function	Inputs	Output	Explanation	Falcon 9 Stage 1 / 2	Saturn V Stage 1
getDeltaV	v_e, m_0, m_f	ΔV	Calculates ΔV	7,979.66 m/s	7,171.81 m/s
				11,082.13 m/s	
getNecessaryFuel	$v_e, \Delta V, m_f$	Wet mass of rocket in m_f units	Calculates fuel required to reach specific ΔV	456 tonnes	2,077 tonnes
				111.5 tonnes	
getTotalDistanceTraveled	$v_e, \Delta V, m_0, R$	Distance traveled by rocket	Calculates total distance traveled while rocket is burning. Supports numerical approximation or exact solution.	412,931.91 m	352,342.32 m
				1143217.91 m	
twoStageRocketDeltaV	$v_e, m_0, m_f \times 2$	ΔV	Uses getDeltaV twice and combines both ΔV values to approximate two stage rocket total ΔV.	12,765.39 m	N/A
getCanReachOrbit	v_e, m_0, m_f	True/False	Determines if rocket can reach orbit	True	False
				True	
getMaximumBurnTime	R, fuel amount	Maximum b_t	Calculates max burn time of rocket	162 seconds	168 seconds
				385 seconds	

Modeling Rocket Mass vs. Velocity

- Simulated increasing payload mass of Falcon 9
- Started with dry mass of 27.2 tonnes
- Mass of rocket was increased by increments of five tonnes
- As payload mass increases, the maximum ΔV decreases and rocket travels less distance ΔX while its engines are burning propellant

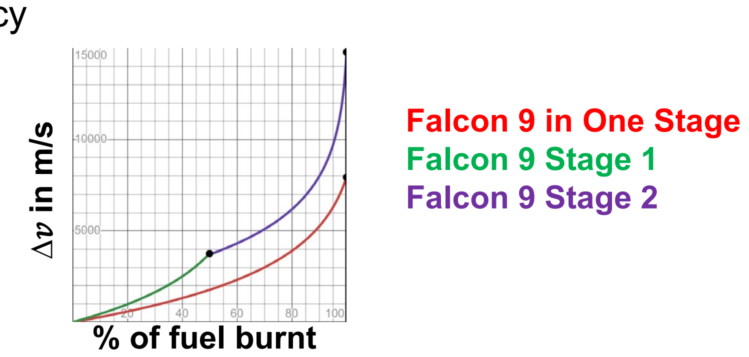




Additional Mass in Tonnes

Results

- Implemented a program to model rocket launches using Java programming language
- Given the characteristics of a rocket, the program can compute eleven attributes pertaining to a rocket launch such as maximum velocity, distance traveled, required fuel, etc.
- Can compare one-stage and two-stage rockets in terms of efficiency



- Orbital rockets with two stages obtain a higher Δv than rockets with one stage, despite having the same amount of thrust and mass as a single stage rocket.

Limitations

- Tsiolkovsky's Rocket Equation doesn't account for several factors (e.g., air resistance, gravity, variable throttle)
- Several assumptions about rockets were made when computing data, for example:
  - Assuming thrusters fire at 100% throttle constantly
  - Assuming the rocket can hold all the fuel necessary

Future Work

- Add function to calculate mass of fuel and oxidizer storage, and adding the mass of the storage container into the dry mass of rocket (i.e., fuel tank mass)
- Implements a function that can compute the amount of fuel needed for the first 1st stage to land after ejecting
  - Can also calculate the amount of thrust needed to land first stage without crashing into the ground

Conclusions

- Rockets deliver important payloads to orbit and beyond.
- By implementing mathematical models with software, it is possible to quickly calculate important characteristics of a rocket launch, given several assumptions about the launch, and ignoring several key factors, like wind resistance and effects of gravity.
- In this project, a Java program was implemented to compute launch values and compare them against real world data.

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