Modeling Rocket Launch Physics with Programming

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Abstract

In 2023, rockets delivered 211 payloads into orbit and computer programs were used in preparation of these launches. A program that implements Tsiolkovsky's Rocket Equation, and related equations, to rocket launches was developed. Given inputs representing the characteristics of a rocket, the program calculates outputs, such as the rocket fuel's burn time, distance traveled, current and maximum velocity, and other characteristics. Data from Falcon 9 and other rocket launches will be used to demonstrate and compare the accuracy of the model.

I. Introduction

There are a multitude of factors that need to be accounted for when using rockets to launch objects. The amount of fuel required for launch, size, and shape of the rocket, and even the location of the rocket launch is considered and questioned before the rocket even begins to be developed. Each of these questions has an answer, and mathematical equations can determine the necessary requirements for the mission. Whether or not the rocket achieves the goal its designed to do depends on the formulas used, and the accuracy of these equations [1].

Despite the existence of technologies that can calculate these values, the goal of the project described in this paper was to learn and understand the underlying theory used by these technologies. "It's not rocket science" implies that a task is simple and easy to understand, due to it not being rocket science, which is known to be complicated and difficult for the average person. This project combined rocket science with computer science to understand the physics behind a rocket launch and the mathematical computations used to make these launches possible.

In this project, a software system implemented in Java was used to implement equations that model different aspects of a rocket launch, such as finding the maximum velocity of a rocket and comparing the efficiency of two Stage rockets. Comparisons between SpaceX's Falcon 9 and Saturn V rockets were made, and the effects of an increased payload are illustrated.

II. Background

A. What is a rocket?

Rockets are vehicles that utilize jet propulsion to move in a specified direction, typically upwards [1]. One type of rocket is a solid-propellant rocket, which uses mixtures of chemicals to propel themselves upward, like gunpowder, or any other fuel and oxidizer that produces an explosion. Another type of rocket is a liquid-propellant rocket, which utilizes liquid rocket fuel with an oxidizer to produce continuous thrust. Liquid-propellant rockets will be explained in further detail in Section II B. Solid-propellant rockets are typically used for missiles and fireworks rather than Space missions, due to their inability to stop burning after starting, but can be attached to liquid rockets to increase the power of the thrusters [2].

B. The Inner Workings of a Rocket

In liquid rockets, rocket fuel and oxidizer mix within a reaction chamber to propel the rocket in the opposite direction [3]. Due to the thrusters facing downward, the rocket goes upward, moving the entire rocket and its payload away from the Earth, against gravity, while the propellant moves downward. The rocket propellant leaves the rocket, and the rocket's velocity (speed in a given direction) will continue to increase so long as fuel is being spent. The objective of a rocket launch is to move upward so the rocket can put objects into orbit or beyond. This process needs to be modeled before the rocket can be launched. One of the most crucial issues is the amount of fuel needed for the rocket, due to the fact that the fuel on the rocket has mass, which will reduce the effective thrust of the rocket. If the amount of fuel is only enough to grant the rocket the desired velocity, then the rocket won't be able to reach orbit due to the distance from Earth being too small. For example, if a rocket reaches its maximum velocity at 10 meters, the rocket won't be successful in getting to orbit. Likewise, if the amount of fuel is too substantial, the rocket won't be able to reach orbital velocity. There are also restrictions on the actual size of the rocket itself. One approach would be to put enough fuel so the rocket would eventually reach the desired height and velocity, but it would become exponentially expensive and large to the point where it would be impossible to construct the rocket.

C. Orbital Velocity

The rocket's goal is to reach orbit, which is a specific height above the core of the Earth, while moving at a certain speed, which allows for an object to continuously move for an unlimited amount of time, if the object remains within the range. The object staying within Earth's orbit is typically a man-made satellite, which can return information back to Earth. The velocity required for an object to reach orbit is defined by Equation 1. *G* represents the gravitational constant. M defines the mass of the entire celestial body, and R represents the radius of the orbit from the center of the celestial body [4].

$$V_{orbit} = \sqrt{\frac{GM}{r}}$$

Equation 1: Orbital Velocity equation

D. Tsiolkovsky's Rocket Equation

Konstantin Tsiolkovsky was a Russian Scientist who derived an equation for calculating the maximum change in velocity of a rocket after a certain amount of fuel has been spent [5]. The Equation is interchangeably called *Tsiolkovsky's Rocket Equation* or the *Ideal Rocket Equation*. The equation can be applied to orbital maneuvers in space and is commonly used for liftoffs. Tsiolkovsky derived the rocket equation by applying Issac Newton's Laws of Motion to a body expelling mass at high speeds in a direction. This allowed him to create the equation without constructing a rocket to validate the accuracy [6].

Despite this, the equation does not factor in several key elements. Air resistance generated from the movement of the rocket through the air is not accounted for, as well as the impact of gravity on the entire mass of the rocket. The Ideal Rocket Equation also does not factor in varying throttle of the thrusters. The equation assumes that the thrusters are firing at maximum power constantly, until all fuel is spent. The equation also assumes that no fuel is left over by the time the rocket finishes thrusting. The simplest form of the equation only requires three defined values to determine the maximum change in velocity, but there are more elements that can be factored in when expanding the equation.

In Equation 2, ΔV represents the maximum change in velocity. V_e represents the effective exhaust velocity of the rocket, which is determined by the strength of the thrusters, after the effects of gravity alter the mass flow rate. The symbols m_0 and m_f represent the initial mass of the rocket, and the dry mass of the rocket, respectively.

$$\Delta v = v_e \cdot \ ln\frac{m_0}{m_f}$$

Equation 2: Tsiolkovsky's Rocket Equation

III. Research and Implementation

This project was formulated around the Ideal Rocket Equation and its variations, which change depending on launch values and situations. A launch taking place on a celestial body other than Earth will have different values for most, if not all, variables in the equations.

After substituting specific values for Tsiolkovsky's Rocket Equation, the equation gives the maximum change in velocity, otherwise known as ΔV. ΔV determines whether the rocket can reach orbit, which depends on the mass to fuel ratio. The mass to fuel ratio is determined by m_0 and m_f, and then inserted into the natural logarithm function, and then multiplied against V_e, which is the effective exhaust velocity of the rocket's thrusters. The natural logarithm of the mass-to-fuel ratio represents the change in mass over time of a rocket burning its fuel, which will be discussed later in this paper. The variable m_i represents the mass of the rocket at time *i*, with index *i* being the number of seconds after the rocket has launched. m_0 implies that the rocket has not launched yet, and that the rocket has not spent any fuel. At time *f*, m_f represents the final mass of the rocket after all the fuel has been spent [5] .

Effective exhaust velocity is calculated by multiplying the standard of gravity (g_0) of the thruster's current altitude against the thruster's Specific Impulse (I_{sp}). Specific Impulse of a thruster is measured in seconds to measure the efficiency of the thrust produced by an engine per unit of mass [7]. In this project, some of the data obtained from other sources only contained the specific impulse rather than the effective exhaust velocity, which led to the effective exhaust velocity being discovered through the equation below.

$$\boldsymbol{v_e = I_{sp} \cdot g_0}$$

Equation 3: Effective Exhaust Velocity

In the project, Java, a programming language, was used to generate outputs for the equations involving Tsiolkovsky's Rocket Equation. The fundamental form of Tsiolkovsky's Rocket Equation (Equation 2) was converted into a method, which served as the starting point for the several other equations that would later be implemented. The method developed in Java requires three values to be sent into the method, substitutes those values for their respective variables, performs the calculations as the equation, and then returns the value. The three required values in the code below are represented as "doubles" which is a data type used to represent floating-point decimal numbers. Block 1 shows the getDeltaV method.

```java
public static double getDeltaV(double Ve, double m0, double mf) {

    double theRatio = (m0/mf);
    double lnFunction = Math.log(theRatio);
    double it = Ve * lnFunction;
    return it;
};
```

Code Block 1: getDeltaV Method

$$\Delta \boldsymbol{v} = \boldsymbol{v_e} \cdot \ \boldsymbol{ln}\frac{\boldsymbol{m_0}}{\boldsymbol{m_f}}$$

Equation 2: Tsiolkovsky's Rocket Equation

E. Real World Application 1: Falcon 9

After the method was created, characteristics of a rocket launch publicly provided by SpaceX were inserted into the program to test the accuracy of the model. The specific model used was the estimated values of the first stage of the Third Block of Falcon 9's Full Thrust rocket, which uses 9 Merlin Engines [8]. The total mass of the rocket was 438.2 tonnes (metric tons, 1000 kg) of fuel, empty mass was 27.2 tonnes, and the Specific Impulse of the rocket at Surface Level was 283 seconds. Due to the effective exhaust velocity not being given, the calculated value was 2773.4 meters per second. The maximum change in velocity given by the program after plugging in the necessary values were similar to the velocity required to reach orbit, which will be discussed later on in the paper .

F. Fuel Over Time

The mass-to-fuel ratio of the rocket mentioned in Section III E has a large impact on the rocket's thrusting capabilities. Tsiolkovsky's Rocket Equation does not factor in burn time of the rocket, which means the maximum change in velocity does not relate to the distance the rocket has traveled [9]. Through the Ideal Rocket Equation, the maximum change in velocity of a rocket with 400 tonnes of fuel, can be the same as a rocket with only 1 metric ton of fuel, depending on the mass to fuel ratio. The difference between the previous two examples would be the duration of the fuel's burn, due to the increase in fuel will be substantially greater than the rocket with much less fuel to burn.

G. Calculating Burn Rate

The burn time of a rocket is dependent on the burn rate of the rocket's fuel. Without a value for the burn rate of a rocket, the burn time of the rocket will be unknown. The data provided by SpaceX for the previous example does not list the burn rate of the rocket's fuel. One possible reason why this information is not available is because thrusters are usually being throttled, rather than firing off at maximum efficiency constantly [10]. Due to this, the value of the burn rate has to be assumed through alternative values given by SpaceX. SpaceX gives the amount of propellant for the designated part of the rocket we are focusing on, as well as the total amount of time that the rocket burned for. Dividing the amount of fuel by the burn time of the rocket gives us a rough estimate of the burn rate of the rocket, which we can now use to determine another variable by modifying Equation 2. In Equation 3, the bottom part of the mass-to-fuel ratio is modified, so the final output of the variable is the change in maximum velocity after a specific amount of time has passed.

$$\boldsymbol{changeInVelocityAtTime = v_e \cdot \ ln\frac{m_0}{m_0 - (burnRate\ \cdot time)}}$$

Equation 3: Change in Velocity at a Specific Time

```
public static double idealRocketEquationAtTime(double exhaustVelocity,
        double initialMass, double burnRate, double time) {
    double mf = initialMass - (burnRate * time);
    double deltaV = getDeltaV(exhaustVelocity, initialMass, mf);
    return deltaV;
};
```

Code Block 2: idealRocketEquationAtTime method (Equation 3)

H. Determining Orbit

As stated above, a rocket needs to burn for a certain amount of time and has a minimum velocity required to reach the orbit of the planetary body it currently occupies. The rocket introduced in Section III E (Stage 1 of Falcon 9 Block 3's Full Thrust) has a maximum change in velocity of 7,708.55 meters per second when plugged into the program. Despite this, the required ΔV to reach a 180-kilometer orbit around the Earth is about 7,800 meters per second [11]. We can determine from this that the rocket does not reach orbital velocity. However, this is only for the first stage of the rocket launch. Some rockets have a second stage, which is intended to get the rocket to its desired location after the rocket has already spent all of the fuel in the first stage of the rocket launch.

I. Multi-Stage Rockets

Having several stages to a rocket launch increases the efficiency of a rocket launch by allowing for a separate, second launch after the rocket has already traveled a certain distance. The first stage of a rocket disconnects from the second stage of the rocket, releasing all the mass from the first stage (fuel tanks, engines, etc.) to allow the second stage to move at a greatly increased thrust capacity [12]. The rocket mentioned in Section III E has a maximum velocity that is about 300 meters per second below the required threshold to reach orbit. Therefore, a second stage firing at a lesser thrust, combined with the release of most of its dry mass, will lead to the rocket reaching the required ΔV to reach orbit.

Figure 1 shows a two-stage rocket used by SpaceX, with the Second Stage having less mass than the first stage of the rocket. The interstage of the rocket connects the first stage of the rocket with the second stage of the rocket. Code Block 3 is pseudocode for calculating the ΔV of a two-stage rocket, by using Tsiolkovsky's Rocket Equation twice and combining the ΔV of the first stage with the second stage's ΔV. Figure 2 is shows a two-stage rocket releasing the first stage and firing the second stage as the first stage releases.

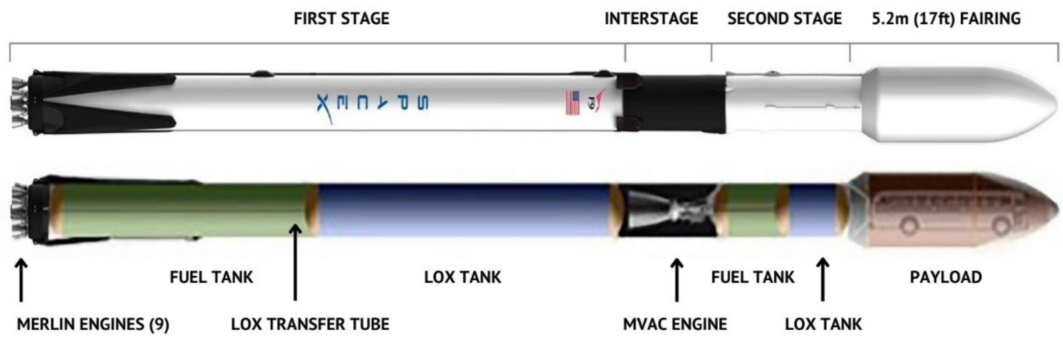

Figure 1: Two Stage Rocket Diagram [13]

```
Set Mass of Entire Rocket = Mass of Stage 1 + Mass of Stage 2

Set DeltaV of the first Stage = getDeltaV(exhaustVelocity1, Mass of Entire Rocket,
[Empty Mass of Stage 1 + Stage 2 Mass])

Set DeltaV of the second stage = getDeltaV(exhaustVelocity2, Mass of Stage 2,
Empty mass of Stage 2)

Set Total Delta V = Delta V of the first Stage + DeltaV of the second stage

Return Total Delta V
```

Code Block 3: Pseudocode of the twoStageRocket Method

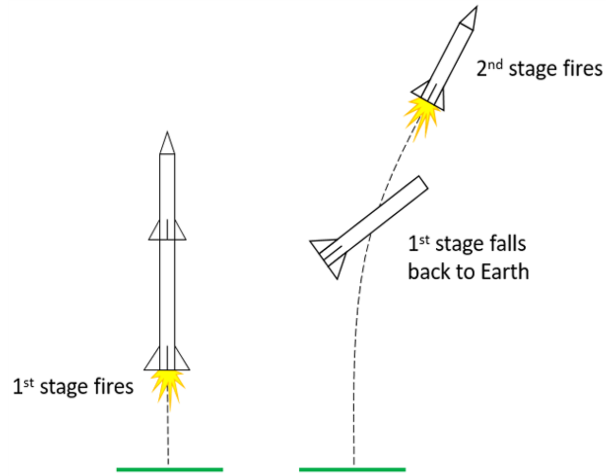

Figure 2: Two-Stage Rocket Launch [14]

J. An Equation for Distance Traveled

Through the equations and methods implemented above, the program was able to calculate the ΔV of the rocket depending on specific characteristics. Tsiolkovsky's Rocket Equation does not compute the distance traveled by the rocket. However, the equation can be used in the calculation of the distance. Velocity, as well as speed, is displacement over time [15], which means a program could be implemented to calculate the distance traveled.

By finding the antiderivative of Tsiolkovsky's Rocket Equation, you can determine the distance traveled. Equation 4 represents Tsiolkovsky's Rocket Equation as an integral. The program implements two methods for computing this distance. The first method was to use trapezoidal approximation to estimate the total distance traveled.

$$\Delta v = v_e \cdot \int_{m_f}^{m_0} \left(\frac{1}{m}\right) dm$$

Equation 4: Integral Form of Tsiolkovsky's Rocket Equation

Trapezoidal approximation calculates the current velocity at each interval, the distance traveled while the rocket moves at the specified velocity, and sums the distances traveled as the rocket's velocity increases. As the number of trapezoids (intervals) used to estimate the model increases, the total distance traveled will approach the exact distance traveled. In Figure 3, the total distance traveled is the area underneath the curve (shown as a black line). As the number of red trapezoids increases, the total distance traveled becomes more accurate.

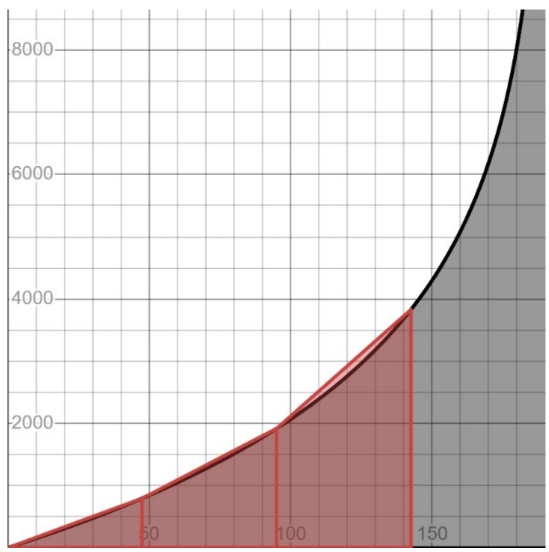

Figure 3: Example Graph to Illustrate Trapezoidal Approximation

The second method to determine the total distance traveled by the rocket involved solving the integral algebraically, resulting in an equation similar to the Ideal Rocket Equation that factors in burn time and burn rate of the rocket [16]. Equation 5 is the equation resulting for this process, with ΔX representing total distance traveled, b representing the burn rate of the rocket, and t representing the time after launch in seconds.

$$\Delta x = V_e\left[t + \left(\frac{m_0}{b} - t\right)\ln\left(1 - \frac{bt}{m_0}\right)\right]$$

Equation 5: Exact Total Distance Traveled Equation

K. Program Implementation

The trapezoidal approximation and the total distance traveled equation mentioned in Section III J were implemented as functions. Creating the methods in Java allowed for changes in parameters to be implemented quickly without having to modify the equation or code. Other methods that were implemented include a method for determining whether a rocket can reach orbit, the amount of fuel necessary to reach a specified ΔV, and to calculate the orbital velocity of a celestial body, depending on the mass of the body and radius of the orbit. Code Blocks 4-6 illustrate the implementation of some of these methods.

```java
public static double getDistanceWithTrapezoids(int a,
        int b, double n, double exhaustVelocity,
        double initialMass, double burnRate) {
    System.out.println("Sent values: \n" + "a: " + a + " b:" + b + " n: " + n);
    double cumulativeEndpoints = 0;

    double deltaX = (b - a) / n;

    //first part
    double firstEndpoint = idealRocketEquationAtTime(exhaustVelocity, initialMass,
            burnRate, a);
    cumulativeEndpoints = cumulativeEndpoints + firstEndpoint;
    //second part
    //from i != b to i !=n+1
    for (double i = (a + deltaX); i < b ;i+=deltaX) {
        double twiceEndpoint = idealRocketEquationAtTime(exhaustVelocity, initialMass,
                burnRate, i) * 2;
        cumulativeEndpoints = cumulativeEndpoints + twiceEndpoint;
    }
    //third part
    double lastEndpoint = idealRocketEquationAtTime(exhaustVelocity, initialMass, burnRate, b);
    cumulativeEndpoints = cumulativeEndpoints + lastEndpoint;
    double halfDeltaX = deltaX/2;
    double areaUnderCurve = halfDeltaX * cumulativeEndpoints;
    System.out.println("Final area under curve: " + areaUnderCurve);
    return areaUnderCurve;
}
```

Code Block 4: getDistanceWithTrapezoids method

```java
public static double getTotalDistanceTraveled(double exhaustVelocity, double deltaV,
        double startingMass, double burnRate
        ) {
    double factorOne = (exhaustVelocity * startingMass) / burnRate ;
    double negativeDeltaV = deltaV * -1;
    double part = java.lang.Math.exp(negativeDeltaV/exhaustVelocity);
    double secondPart = (1 + deltaV/exhaustVelocity);
    double bothPartsCombined = part * secondPart;
    double factorTwo = 1 - bothPartsCombined;
    double product = factorOne * factorTwo;
    return product;
}
```

Code Block 5: getTotalDistanceTraveled method

```java
public static double getNecessaryFuel(double Ve, double deltaV, double mf) {

    double theRatio = (deltaV/Ve);
    double eFunction = Math.exp(theRatio);
    double oneFactor = eFunction - 1;
    double finalAnswer = mf * oneFactor;
    return finalAnswer;
}
public static void testGetMassFlowRate() {
    double grav = 9.8;          //standard gravity
    double isp = 283;           //merlin engine specific impulse in seconds
    double thrust = 7601370;    //in newtons
    double massFlowRate = getMassFlowRate(isp, thrust, grav);
    System.out.println("Mass Flow rate of engine: " + massFlowRate);
}
```

Code Block 6: getNecessaryFuel method

IV. Results and Limitations

A. Falcon 9 and Saturn V Example

The two stages of SpaceX's Falcon 9 Rocket and the first stage of Saturn V, the rocket which brought the first person to the moon, were used to test the program. Table 1 shows the characteristics of these rockets and Table 2 shows the outputs of the program based on these characteristics.

	Falcon 9 Stage 1 / Stage 2	**Saturn V Stage 1**
Fuel / Oxidizer	RP-1 – LOX	RP-1 - LOX
Specific Impulse, Effective Exhaust velocity	283 s, 2,773.4 m/s	263 s, 2,577.4 m/s
	348 s (v), 3,410.4 m/s	
Dry Mass	27.2 tonnes	130 tonnes
	4.5 tonnes	
Wet Mass	483.2 tonnes	2,214 tonnes of fuel
	116.0 tonnes	
b_t/R	162 seconds / 2.537037 tonnes/s *	168 seconds / 12.40476 tonnes/s
	397 seconds / 0.289 tonnes/s *	

Table 1: Real World Rocket Data

Example Function	Inputs	Output	Explanation	Falcon 9 Stage 1 / 2	Saturn V Stage 1
getDeltaV	v_e, m_0, m_f	ΔV	Calculates ΔV	7,979.66 m/s	7,171.81 m/s
				11,082.13 m/s	
getNecessaryFuel	v_e, ΔV, m_f	Wet mass of rocket in m_f units	Calculates fuel required to reach specific ΔV	456 tonnes	2,077 tonnes
				111.5 tonnes	
getTotalDistanceTraveled	v_e, ΔV, m_0, R	Distance traveled by rocket	Calculates total distance traveled while rocket is burning. Supports numerical approximation or exact solution.	412,931.91 m	352,342.32 m
				1143217.91 m	
twoStageRocketDeltaV	v_e, m_0, m_f × 2	ΔV	Uses getDeltaV twice and combines both ΔV values to approximate two stage rocket total ΔV.	12,765.39 m	N/A
getCanReachOrbit	v_e, m_0, m_f	True/False	Determines if rocket can reach orbit	True	False
				True	
getMaximumBurnTime	R, fuel amount	Maximum b_t	Calculates max burn time of rocket	162 seconds	168 seconds
				385 seconds	

Table 2: Output for Falcon 9 and Saturn V

B. Falcon 9 Launch Configurations

Two comparisons were made with the data gathered from the program. First, the efficiency of multi-stage rockets was evaluated. Second, the effects of adding larger payloads were analyzed. Figure 4 highlights the benefits of having a multi-stage rocket, displaying the higher ΔV from using a rocket with two stages, as opposed to creating a rocket with only one stage. Figure 5 represents the change in ΔV and ΔX because of increasing the payload mass in a rocket by a specified interval. The chart shows that as the payload, or dry mass, of the rocket increases, the ΔV and ΔX of the rocket decreases.

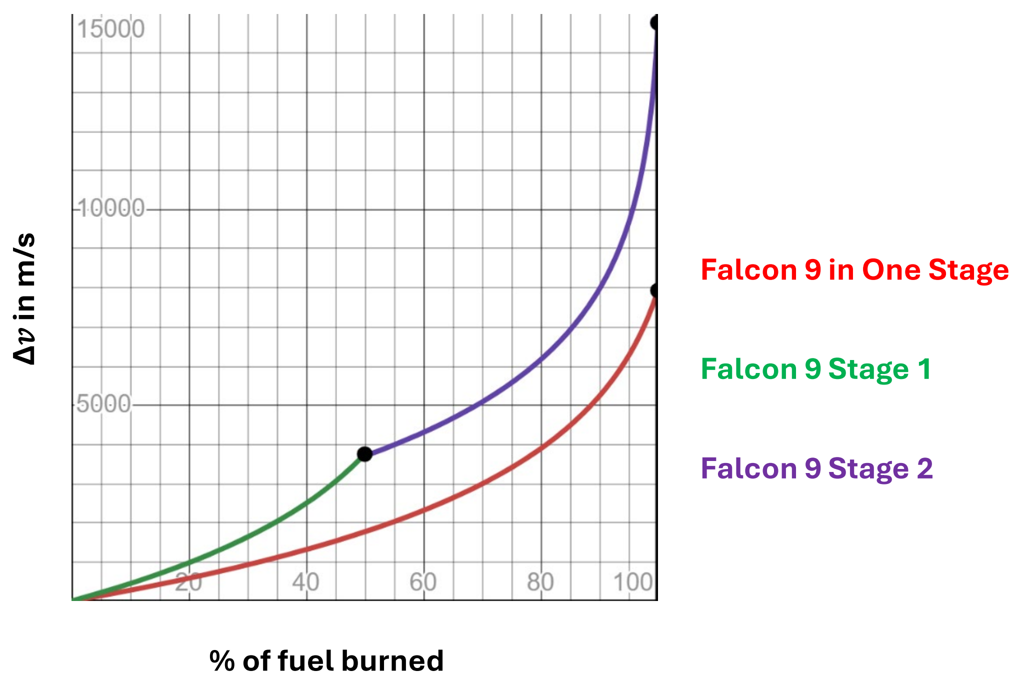

Figure *4: One-Stage and Two Stage Rocket Comparison*

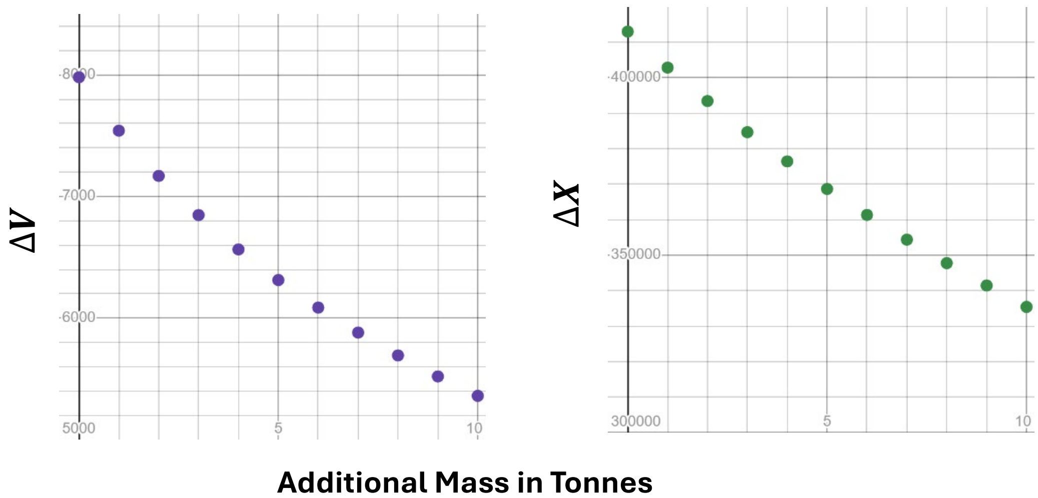

Figure 5: Increasing Payload Mass by increments of 5 tonnes

C. Limitations

As mentioned in Section II D, Tsiolkovsky's Rocket Equation, the foundation of the project, makes several assumptions about rocket launches, as well as not accounting for factors that heavily impact a rocket's launch. Air resistance effecting the rocket and effects of gravity on the rocket are not considered within the project. A varying throttle is also not considered within the project, and neither is the propellant holding capacity for the rocket itself. The equation also assumes that as you add fuel, the rocket can hold any additional fuel required without increasing the mass of the fuel tanks.

V. Future Work

One potential addition to the project would be implementing the mass of the propellant storage of the rocket, and factor it into the equation as part of the dry mass. As mentioned in Section IV C, the dry mass of the storage container is not considered when adding additional fuel to the rocket. Another possible addition could be implementing a function that models saving a specific amount of fuel necessary for the first stage of a rocket to land, and calculate the thrust needed to safely land.

VI. Conclusion

In this project, a program that was able to calculate different values for a rocket launch was implemented in Java. To implement this project, it was necessary to do extensive research on the physics and mathematical modeling of rocket launches. Programming the values allows for quick substitution of specific values to produce different outputs easily. Modeling rocket launches is a crucial and necessary part of every rocket mission.

Acknowledgements

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