Differences in learning geometry among high and low spatial ability pre-service mathematics teachers

Hasan Unal*, Elizabeth Jakubowski and Darryl Corey

*Mathematics Department, Yildiz Technical University, Davutpasa Campus, Esenler, Istanbul, 34210 Turkey; bSchool of Teacher Education, Florida State University, 0215 MCH, Tallahassee, 32306-4490 United States; cDepartment of Mathematics and Statistics, Kennesaw State University, 0205MCH, Kennesaw, 30144-5591 United States

(Received 19 July 2008)

The objective of this study was to investigate and characterize the geometric thinking and understanding of four pre-service middle and secondary mathematics teachers while considering their spatial ability levels. To investigate the differences, if any, that existed among these pre-service middle and secondary teachers with different spatial ability levels and understanding geometry, pre- and post-test designs were employed using Mayberry’s (J. Mayberry. An investigation of the Van Hiele levels of geometric thought in undergraduate pre-service teachers, Unpublished Doctoral dissertation, University of Georgia, Athens, GA, 1981) protocol. Four contrasting cases in terms of spatial ability scores are examined using the van Hiele model to provide a description of geometric understanding. The study participants were chosen using the Purdue visualization of rotations test (ROT; G.M. Bodner and R.B. Guay, The Purdue visualization of rotations test, Chem. Educ. 2 (1997), pp. 1–18) from among a pool of pre-service middle and secondary mathematics teachers at a major research university in the southeast United States. The results were supportive of previous research in this area. Learners with low spatial abilities are more challenged geometry learners. While learners identified with mid-range spatial abilities showed the most change in van Hiele levels after instruction, the low spatial ability student showed the least amount of change. The student with high spatial ability showed some change after instruction. It was identified that instructional activities that afford opportunities for fostering spatial abilities must be included in pre-service programmes so that future teachers have a mathematical foundation from which to teach geometry.

Keywords: spatial ability; van Hiele model; pre-service mathematics teachers

1. Introduction

Since 1959 the International Association for the Evaluation of Educational Achievement (IEA) has conducted international comparative studies of the mathematics and science performance of students. One series of studies focused on mathematics: First International Mathematics Study (1963–1967), Second International Mathematics Study (1977–1981), Third International Mathematics Study (1995–1998), and most recently, the Fourth International Mathematics and Science Study (1999–2003). These studies have shown that there is a strong correlation between spatial ability and mathematical achievement.
and Science Study (1993–1997), Third International Mathematics and Science Study-Repeat (1997–2001) and Trends in Mathematics and Science Study (2001–2004). These studies provide a picture of United States (US) students’ mathematical achievement. While there have been increases in achievement at different grade levels, the mathematics achievement of US students continues to lag. In TIMSS 2003, the US eighth grade average (504) was above the international average (467). Of the five mathematical areas evaluated geometry was the lowest for US eighth grade students (472) and data the highest (527). While overall the US was in the top third, in geometry the US was in the bottom half for eighth grade students. This trend was also evident for fourth grade US students [1]. A question that emerges as one reads the reports is why does US student performance in geometry lag behind when compared to other content areas and students’ peers in other countries? The source of the answer could reside in a number of explanations. This study posits that US mathematics teachers are not successful in engaging their students in appropriate geometry instruction. ‘Opportunity to learn’ was found in the First International Mathematics Study [2] to be a good predictor of differences found in student performance. Thus, potential learning opportunities may abound for students but teachers whose geometric knowledge and/or spatial ability is limited may not have the capacity to make adjustments to curriculum to address the needs of students with varying learning needs. What the field has is limited understanding of the relationship between van Hiele levels and spatial ability. In particular, research on the relationship between these two for teachers who are the instructional leaders making instructional decisions in the classroom.

Research in spatial ability and the van Hiele model were historically treated as separate domains until 1997. At that time a connection was made by Saads and Davis [3] with a study that investigated the relationship between spatial ability and the van Hiele levels. Their work focused on three-dimensional geometry. According to Henderson [4] how students learn geometry greatly depends on the teachers and how they make instructional decisions at critical moments in the classroom.

Mathematics educators have been studying van Hiele levels [3–10] and spatial ability [3,11–27] for many years. These two areas have long been of major importance to mathematics educators because of the potential they have for improving both mathematical understanding and pedagogy. This is especially true in the discipline of geometry [4,9,28–31].

2. The purpose of the study
The purpose of this study was to investigate and characterize the geometric thinking and understanding of four pre-service middle and secondary mathematics teachers while considering their spatial ability scores. This study was conducted using members from an undergraduate informal geometry course for pre-service teachers at a major research university in the southeast United States. Using multiple sources of qualitative data – audiotapes of interviews with students, transcripts of those tapes, classroom observations and researchers’ field notes, worksheets of case study students, and students’ artefacts – case study analyses were undertaken to identify patterns and changes in students’ geometric thinking with respect to their understanding. This research project followed the model of case study (multiple cases) set forth by Creswell [32]. Creswell [32] defines a case study as an exploration
of a ‘bounded system’ or a case (or multiple cases) over time through detailed, in-depth data collection involving multiple sources of information rich in context.

3. Research questions
The question that was addressed in this study was:

Do the spatial abilities of perspective mathematics teachers affect their understanding of geometry? More specifically, what differences, if any, exist among these four prospective middle and secondary mathematics teachers with different spatial ability scores and their understanding of geometry?

This question is important because it may be a means for providing a partial explanation for why US students’ geometry performance remains far below the performance of most of their international peers.

4. Theoretical framework
For more than 20 years, the van Hiele model of geometric thought has served to characterize individuals’ thinking in geometry [4,10]. The van Hiele model of geometric thought emerged from the works of two Dutch mathematics educators, Dina van Hiele-Geldof and Pierre M. van Hiele at the University of Utrecht. The van Hiele model consists of five levels, that is, the authors suggested that geometric thought develops in sequence of five levels. These levels, as ranged from the lowest to the highest, include: Level 0-visualization, Level 1-analysis, Level 2-informal deduction, Level 3-formal deduction and Level 4-rigour [5]. Burger and Shaughnessy [5, p. 31] describe the levels of thought as they apply to geometric development as follows:

- **Level 0 (Visualization):** The student reasons about basic geometric concepts, such as simple shapes, primarily by means of visual consideration of the concept as a whole without explicit regard to properties of its components.
- **Level 1 (Analysis):** The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established.
- **Level 2 (Abstraction):** The student logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of set of properties in determining a concept.
- **Level 3 (Deduction):** The student reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions and theorems.
- **Level 4 (Rigour):** The student can compare systems based on different axioms and can study various geometries in the absence of concrete models.

Differences in numbering the levels as 0–4 or 1–5 abound in the literature. While the numbers may differ the description of what one is able to do at the first, second, third, fourth or fifth level remains the same. The van Hieles themselves labelled the levels 0–4. For the purposes of this study the use of 1–5 will be used.

The van Hiele model not only informs us about the learning of geometry but also about the teaching of it. Van Hiele [33, pp. 53–54] described five stages which, when
encountered in the process of instruction, lead to a higher level of thought. The first stage is information where pupils get acquainted with a working domain. The second stage is guided by orientation; students are guided by a task (given by the teacher or made by them) with different relations of the association that has to be formed. In the third stage, which is explanation, they become conscious of the relations, they try to express them in words and they learn the technical language accompanying the subject matter. In stage four, free orientation, they learn by general tasks to find their own way in the network relations. Lastly in the fifth stage, integration, students build an overview of all they have learned on the subject, of the newly formed network of relations now at their disposal. Although the van Hiele levels of geometric thought were first proposed in 1957, some 50 years ago, they are still widely accepted by the mathematics education community today. In fact, there is currently no other model or framework for geometric thought that is more widely accepted in the mathematics education community.

5. Key concepts
Spatial ability is defined and evaluated in many ways in the literature; therefore, it is difficult to put forward one precise definition for the concept. However, a routinely used definition states spatial ability as the ‘generation, retention, retrieval and transformation of visual images’ [34, p. 904]. For the purposes of this study the two types of spatial ability considered are spatial orientation and spatial visualization. They are defined as follows:

- **Spatial orientation** [35, p. 1435]: ‘The spatial orientation factor has been described as a measure of the ability to remain unconfused by changes in the orientation of visual stimuli, and therefore it involves only a mental rotation of configuration.’
- **Spatial visualization** [35, p. 1435]: ‘The spatial visualization factor measures the ability to mentally restructure or manipulate the components of the visual stimulus and involves recognizing, retaining and recalling configurations when the figure or parts of the figure are moved.’

6. Literature review
The mathematics education community has been studying both van Hiele levels and spatial ability as separate research domains for some time now [4,12,13,20].

Recently research [3] has shown that these two areas, spatial ability and van Hiele levels of geometric thought, are essential for both students and teachers because of their potential for improving both mathematical comprehension and pedagogy.

6.1. Van Hiele model
The first research on the van Hiele model, conducted after the van Hieles proposed their model in 1957, was done by Soviet educators in the early 1960s. Wirzup formally introduced the van Hiele model in the United States in 1974 with a presentation at the annual meeting of the National Council of Teachers of Mathematics. After that, numerous US researchers conducted studies using this
model [4–6,10,31]. Among the early participants were the Oregon State, Brooklyn College and Chicago projects, which were examples of large scale designed research studies (for details see [4, pp. 26–31]). The work utilizing the van Hiele model has addressed learning geometry by a variety of learners, e.g. K-12 students, post-secondary students, pre-service teachers and practising teachers.

The body of research on pre-service teachers has included studies from the United States [4,10], Spain [7,8] Portugal [36], Britain [3] and China [37]. These researchers studied the van Hiele levels of the reasoning of pre-service teachers on several geometric concepts.

Research related to pre-service teachers and the van Hiele model was first conducted by [10]. This involved an investigation of pre-service elementary teachers’ geometric knowledge based on the van Hiele model. Mayberry’s research was designed to investigate the fixed sequence property of the model and to look at whether an individual would demonstrate the same level of thought in all areas of geometry. Mayberry [38] constructed test items covering seven geometric concepts, which included squares, isosceles triangles, right triangles, circles, parallel lines, similar figures and congruent figures. Items for each concept and level were developed.

Nineteen pre-service elementary teachers were individually interviewed using the revised questions during two 1-hour audio taped interviews. During the interviews pre-service teachers were provided with paper, a straight edge and a pencil. Responses were analysed and supported Mayberry’s hypothesis that the van Hiele levels are hierarchical. However, Mayberry did not find evidence that individuals demonstrated the same level of thought in all areas of geometry.

Henderson [4] used Mayberry’s interview-based instrument to identify five pre-service secondary mathematics teachers’ geometric thinking. Following the interviews, each participant was observed and video taped while teaching five high school geometry students. She conducted stimulated-recall interviews during and following the teaching segment for each pre-service teacher. Henderson [4] found that pre-service teachers’ geometric thinking levels were reflected in their instruction and, as a result, the level of understanding of pre-service teachers influenced students’ difficulty or insight.

Since the 1980s additional researchers have continued to utilize the model as a framework for further study. Gutiérrez et al. [8] conducted a study to evaluate acquisition of van Hiele level of primary pre-service teachers by proposing an alternative paradigm. This research differed from other research works in that instead of assigning students to one level [4,10,31] they looked at degree of acquisition of a given level (e.g. low, intermediate, high acquisitions, etc.) in three-dimensional geometry. Gutiérrez et al. [8] designed a spatial geometry test to evaluate the van Hiele level of students’ thinking in three-dimensional geometry. They identified that acquisition of the lower level is more complete than acquisition of higher level and they also observed that not all students used a single level of reasoning, but some of them showed several levels at the same time. The researchers did not reject the hierarchical structure of the levels, but they pointed out the complexity of the human reasoning process when teachers adapt the van Hiele model to their own instruction. Another interesting result was that students showed better acquisition of Level 3 than of Level 2.

Saads and Davis [3] examined the importance of spatial abilities in the academic development in geometry. They designed a study to investigate 25 pre-service
secondary teachers’ van Hiele levels and spatial abilities and to relate students’ questioning and use of language to both levels. Saads and Davis [3] constructed test items to determine participants van Hiele levels and spatial abilities.

The current literature related to van Hiele that concentrates on the determination of van Hiele levels of pre-service teachers [3,4,8,10] is in relation to their instructional behaviours [4] to their spatial ability [3].

De Villiers [39–41] has pointed out some problems with the van Hiele model and characterization of levels; that is, hierarchical thinking (class inclusion) may not be independent of teaching strategies and might occur at visualization using appropriate teaching strategies and tools (such as dynamic geometry). On the other hand, if class inclusion is not dealt with at the visual level, research evidence seems to indicate that partitional thinking becomes highly resistant to change later on.

6.2. Spatial ability

Another important concept in learning geometry is spatial ability. The first studies in the field of spatial ability date back to the 1940s and 1950s in the mathematics education literature. However, the concept was also of interest to psychologists (for details, see [20]) a decade earlier. In those initial studies, numerous mathematics educators [11,21,22] investigated the relationship between spatial abilities and mathematical abilities in different contexts such as algebra and geometry. They found that spatial ability was correlated more highly with ability in geometry than in algebra (cited in [20, p. 181]). Since then, spatial abilities have been widely accepted as crucial for high mathematical abilities [13–20,23–25,30]. The National Council of Teachers of Mathematics [42] identified the development of spatial abilities as a central goal in school mathematics especially in geometry. Thus, mathematics educators, and psychologists, remain interested in spatial ability today. Spatial abilities and their role in learning is also recognized in chemistry [35,43,44], engineering [45] geosciences [46] and physics [47].

Battista et al. [18] conducted a study to investigate the importance of cognitive development and spatial visualization for pre-service elementary teachers’ achievement in a geometry course, and the effects of instruction type on improving spatial ability. Battista et al. [18, p. 338] found that the spatial visualization scores at the end of the semester were significantly higher than at the beginning of the semester as measured by the Purdue spatial visualization test. Using an experimental design one group of students \( (n=36) \) did not complete any spatial activities, while other participants in the other three courses completed spatial activities designed specifically to improve spatial visualization. The increase in the spatial visualization scores for those who received spatial activities was significantly higher than those who did not. The researchers concluded that the type of activities, such as paper folding, tracing and symmetry, which were used in their study may improve students’ spatial ability. Although this study found that improvements on students’ spatial ability was directly due to specifically designed activities, they recommended future research stating, ‘more research is needed to determine if geometry instruction that emphasizes spatial activities benefits one group of students more than another’.

Brown and Wheatley [48] found that while students with high spatial ability performed average or below average in standardized tests, they showed an advanced understanding of mathematical ideas, and were able to solve non-routine
mathematical problems creatively. Conversely, students with low spatial ability performed at average or above average in standardized tests but they were unable to solve non-routine problems and good reasoning in mathematical tasks as a whole. This finding again highlighted the importance of spatial ability as an important factor for conceptual understanding and giving meaning to a mathematical activity. Also Presmeg [49] found visual–spatial scheme (pattern imagery) an important aspect of problem solving and abstraction.

Battista [12] investigated the role of spatial visualization in the performance of gender differences in geometry and found that male and female students differed in spatial visualization with respect to their performance in high school geometry but showed no differences while applying geometric problem solving strategies. More specifically, spatial visualization was only predictor for females, whereas logical reasoning was the critical factor for males. Battista interviewed both teachers and the second teacher required students to draw diagrams for geometric problem while the first teacher only strongly encouraged this. Although the researcher did not observe the classroom settings, the researcher cautiously concluded that teacher effect and instructional emphasis might play a role in the development of spatial visualization skills.

There is evidence from the studies that suggests that the interaction between geometric knowledge and spatial ability is an important dimension to investigate. The significance of further studies with pre-service teachers is important, in that they will be future instructional leaders in the classroom making instructional decisions regarding the learning opportunities for their students in geometry. As an exploratory study into this area, this study sought to further establish the relationship between geometric knowledge and spatial ability of pre-service mathematics teachers.

7. The study

7.1. Settings and participants

The participants for this study were all members of an undergraduate geometry course, namely MAE 4816 Elements of Geometry. The course is offered at a major research university in the southeast United States. This course was designed for pre-service middle and secondary mathematics education majors and is a required course. There were 28 students in this class, 6 male and 22 female. In terms of ethnicity 3 of the 28 students were African American; 1 male and 2 females, 1 student was Asian and 1 was Hispanic. The remaining students were Caucasian. The physical layout of the classroom was similar to many other classrooms in the US. There were rows of desks, chalk and blackboards, and one computer and projector. Students sat side-by-side in pairs and were lined up in rows facing the chalkboard and stretching to the back of the classroom. Class meetings were held regularly in this setting; however, starting in the fourth week, the class also met in a computerized classroom on many occasions. A total of 26 teacher candidates, 5 male and 21 female, participated in the larger study. Six pre-service teachers volunteered to participate in this study. Among the six, only four pre-service teachers completed both the pre- and post-interviews and test. Of the four participants, two were female and two were male. For future reference and discussion, the respective participants were given the pseudonyms Allen, David, Barbara and Cathy.
participants will be called pre-service middle and secondary teachers because none of them had teaching experience prior to the data collection.

7.2. *Data collection*

The use of a variety of data collection techniques permitted the researchers to establish general principles about the geometric understanding of these four pre-service middle and secondary mathematics teachers.

7.2.1. *Clinical interviews*

Clinical interviews designed to explore the pre-service teachers’ knowledge of geometry were conducted twice; before and after the college geometry course. An intention of clinical interviews is to gain insight into the interviewee’s thinking. It is not the intention to teach but to describe how one approaches a mathematical problem or a question. These interviews concentrated on having the pre-service teachers responding to the protocol designed by Mayberry [10] and used by other researchers [4]. Responses to a series of 62 questions, each printed on one-fourth of letter size paper, were audio taped. The interviewers also recorded responses on a check sheet. Figure 1 shows examples of questions that were similar to those asked at each level.

Participants were provided with blank paper, a pencil and a straight edge. Instructions to participants encouraged the drawing of any necessary diagrams. Participants were also told that the researchers were interested in investigating their thinking during their obtaining of a result; therefore, the reason an answer was given was of as much interest as was the correct or incorrect answer. Participants were encouraged to ‘think aloud’ and to give reasons as they went through the process of getting a result. No probing questions were asked to lead them to correct answers; however, when clarification was needed, the researchers asked and answered some

<table>
<thead>
<tr>
<th>Level I: Recognition</th>
<th>Suppose these two lines will never meet no matter how far we draw them. What word describes this?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level II: Analysis (properties of figures)</td>
<td>What is true about the sides of a square?</td>
</tr>
<tr>
<td>Level III: Ordering, Class inclusions, relations, implications</td>
<td>Square A is sometimes, always, never similar to Rectangle B. Give reasons for your answer. Can a right triangle be isosceles? Why?</td>
</tr>
<tr>
<td>Level IV: Deduction</td>
<td>Figure ABCD is a parallelogram. AB = BC and ( \angle ABC ) is a right angle. Is ABCD a square? Prove your answer.</td>
</tr>
<tr>
<td>Level V: Rigor</td>
<td>What is the difference between an axiom and a theorem?</td>
</tr>
</tbody>
</table>

Figure 1. Examples of interview items aligned with van Hiele levels.
questions for clarification purposes only. Interviews were conducted in a classroom on the university campus.

The pre-service teachers and investigators were the only people present during each interview. Each question was read aloud by the pre-service teacher and then he/she attempted to find an answer. Responses were analysed using criteria established by Mayberry [10]. This criteria did not require that correct answers be given for all items for the level to be achieved but was based on a proportion of correct answers. Thus, a level might be achieved if 6 of 9 answers were correct, or 3 of 4 answers. Each concept at each level did not have the same number of items so the number correct varied (for further details, see [4,10]).

7.2.2. The Purdue visualization of rotations

The Purdue visualization of rotations test (ROT) is one of the elements of the Purdue spatial visualization test battery. The Purdue ROT consists of 20 items that require students to:

1. study how the object in the top of the line of the question is rotated,
2. picture in your mind what the object shown in the middle of the line of the question looks like when rotated in exactly the same manner and
3. select from among the five drawings (A, B, C, D and E) given in the bottom line of the question the one that looks like the object rotated in the correct position [35].

As instructed in the scale put forward by Bodner and Guay [35] students were given 10 min. The researchers explained to the students that this activity was not going to affect their grades in any way. At the beginning of semester each student took the ROT. One point was awarded for each correct response. The total score was the sum of correct responses on the test.

8. Results

This case study of pre-service teachers focused on middle and secondary pre-service teachers enrolled in the same geometry course at a major research university in the southeast United States. The following sections provide a description of the differences in geometric understanding of low, mid-range and high spatial ability as measured by the Purdue ROT and the Mayberry van Hiele protocol.

The Purdue ROT was administered to all 28 students enrolled in the course to determine their spatial ability scores. The range for the class was scores of 3–19, with \( \mu = 12.77 \) and \( \sigma = 4.23 \). Students were classified as having high spatial ability if they were one standard deviation above the mean and low spatial ability if they were one standard deviation below the mean. Seven students in the course were in the high category while five were in the low category. The remaining class members were between one standard deviation below and above the mean (the mid-range spatial ability category). The scores of 11 students were below while scores of 15 students above the mean.

For this study four participants were selected of which two had a score above the mean and two below. Figure 2 summarizes the four pre-service teachers’ spatial ability scores. Allen’s score is at the high end of the range of scores; Cathy at the low
end and Barbara and David had scores near the mean. Thus, Allen was identified as high spatial ability, Cathy low and Barbara and David as medium.

Using the Mayberry van Hiele protocol each participant was assigned one of five levels (visualization, analysis, abstraction, deduction and rigour) reflecting their level of understanding of seven different geometric concepts (squares, right triangles, isosceles triangles, circles, congruency, similarity and parallel lines). The interviews conducted both at the beginning of the course and upon course completion were coded so that each participant was given a beginning and ending van Hiele level for each concept.

8.1. High spatial ability

Allen scored above the class mean on the Purdue ROT and was at the extreme of the scale for spatial ability. Thus, for this group he was categorized with high spatial ability. Figure 3 shows the differences in Allen’s beginning of course interview and end of course interview for each of the seven geometric constructs.

Allen’s geometric thinking was at level III for all the concepts at the beginning of the course. As seen from Figure 3 at the end of the course there was a change in Allen’s levels for six of the concepts. By the end of the course Allen had shown growth in geometric understanding for all concepts except isosceles triangle. For interview items that did not include a drawing Allen was able to construct an image that facilitated his analysis and subsequent response to the item. For example, four items asked about the necessary and sufficient conditions for a figure to be a square, right triangle, isosceles triangle or parallel lines (Figure 4). Allen correctly answered these items in the pre- and post-interviews.

8.2. Mid-range spatial ability

Two of the pre-service teachers, Barbara and David, both scored near the mean of the class on the Purdue ROT. Barbara was below the mean and David was above. David’s growth as reflected by the interviews was not as dramatic as was Barbara’s growth. Figure 5 shows David’s growth and Figure 6 shows Barbara’s.
David’s growth on the concepts of squares and right triangles is not as surprising as is the absence of a change in the concept of congruence. This result may be linked to the fact that instructional activities were more focused on the relationships associated with the concepts of polygons (e.g. square and triangle) and less on congruence. Thus, the choices of learning opportunities by the instructor were not sufficient to allow for the growth in knowledge of this concept. Barbara’s growth was evident in six of the seven topics.

An analysis of the various interview items indicates that the questions used at levels III and IV (see Figure 1 for examples) tended to not include drawings so that the interviewee had to draw a figure based on the given information. The drawings attempted by David and Barbara during the pre-course interview show that they were struggling to draw an appropriate figure to assist with the evaluation of statements that were asking about relationships or class inclusions of concepts.

### 8.3. Low spatial ability

Cathy, the fourth pre-service teacher, had the lowest spatial ability score of the four and in the class. Figure 7 shows her geometric development as measured by the van Hiele test.
Cathy’s lack of change in all but one of the concepts is directly connected to her low spatial ability. Items on the interview for levels I and II almost always had a figure included. The criteria for achieving a level allowed an interviewee to give incorrect responses for some items and still have achieved the level and at these levels. For example, when Cathy was given problems similar to that shown in Figure 4, she was not able to draw an image that would help her to analyse the figure and determine what conditions are necessary. This was observed for the concepts of square, right triangle, isosceles triangle and circle. This was similar to findings in work of [4,10], which showed that when a figure was not provided the pre-service teacher focused on a generalized drawing and did not consider specific drawings as called for in the interview item.

9. Summary of findings

On the basis of the findings of this case study, spatial ability was an important construct for the development of the geometric understanding of these four pre-service teachers. Cathy, whose spatial ability score was the lowest (5) showed minimal to no growth in geometric understanding (Figure 8). Both at the beginning and end of the study, she was classified at a level II or level I geometric understanding in all but one category, circle, where she remained at level III throughout. At the end of the study, Cathy showed improvement in geometric understanding in only one category, similarly, where she moved from level I to level II. It is also important to point out here that when materials and instruction are operating at a higher level than a student’s level of understanding the lack of alignment between these often prevents student growth in understanding as measured by the van Hiele levels. This is a possible explanation for Cathy’s minimal growth in geometric understanding at the end of the study.

Figure 8 also shows that the students with mid-range spatial ability scores, Barbara (11) and David (14), varied in their improvement of geometric understanding. However, they both showed greater improvement than Cathy. Barbara showed the most growth in geometric understanding by moving up at least one level in six of the seven categories (square, right triangle, isosceles triangle, circle, congruence and similarity). Her greatest increases occurred in the categories of square and similarity, where she moved from level II to level IV and from level I to level IV, respectively. Barbara’s overall growth placed her at a level IV understanding in all but two categories, congruence and parallel lines, where she finished at level III. David showed growth in geometric understanding in two categories, square and right triangle, where he moved from a level III understanding to a level IV

Figure 7. Cathy’s geometric development from pre-interview to post-interview.
understanding in both. This growth placed him at a level IV understanding in all categories except congruence where he remained at a level III.

Lastly, the student with the highest spatial ability score, Allen (19), also showed greater improvement than Cathy, the student with the lowest spatial ability score. Allen began the study at a level III geometric understanding in all categories and finished the study with a geometric understanding of IV in all but two categories. In those two categories, isosceles triangle and parallel lines, he remained at level III.

10. Conclusions
On the basis of the results and findings of this study, the spatial ability of these four perspective mathematics teachers’ did affect their understanding of geometry. Those pre-service teachers with high or mid-range spatial ability scores showed both a greater beginning level of geometric understanding and a greater improvement in geometric understanding than the student with the lowest spatial ability score. In fact, the student with the lowest spatial ability score, Cathy, showed almost no growth in geometric understanding throughout the study.

Research has shown that in order to have any degree of success in a high school geometry class necessitates a level III understanding on the part of the student [31]. When the teacher has not achieved this level or level IV, then the instructional decisions and questions asked for students lack the depth needed to ensure some mathematical richness of the problems [4]. This study shows evidence that spatial ability is important in the geometric understanding of pre-service teachers. Further work should explore the nature of the mathematical tasks provided to pre-service teachers so that appropriate tasks that would assist low spatial ability learners progress more in their geometric knowledge in order to have a level of understanding that is reflective of the level IV of the van Hiele model. Another area that is important to examine is the use of geometric reasoning by individuals with differing spatial abilities. The types of understanding represented by levels III and IV of the van Hiele model (e.g. necessary conditions, relationships, class inclusions and proof) rely on an ability to reason through statements of relationships or properties. How important spatial ability is in the process of reasoning in geometry is an area that merits additional work along with Presmeg’s [50] 13 research questions.
References


